Decomposing the Integrated Assessment of Climate Change

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Abstract

We present a decomposition approach for integrated assessment modeling of climate policy based on a linear approximation of the climate system. In our formulation the economic and natural science components are processed independently on different time scales. Turnpike properties of the Ramsey growth model can be exploited to provide a precise representation of post-terminal emissions and to reduce the economic horizon required to accurately approximate transition paths. Germaine to the economic assessment of climate policies, our decomposition accommodates formulation of the economic model in a complementarity format and thereby provides a means of incorporating second-best effects that are not easily represented in an optimization model.

JEL classification: C61, C63, D58, D61

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1 Introduction

Integrated assessment modeling emerged in the mid-eighties as a new paradigm for interfacing science and policy concerning complex environmental issues. An integrated assessment model combines complementary knowledge from various disciplines in order to derive insights into questions of policy design. Integrated assessment models (IAMs) link mathematical representations of the natural system and the socio-economic system to capture cause-effect chains including feedback. An early example of integrated assessment is the RAINS model of acidification in Europe [Alcamo et al. 1985]. Over the past years, a variety of models have been developed for the integrated assessment of climate change – for surveys see Weyant et al. [1996], Parson and Fisher-Vanden [1997], or Kelly and Kolstad [1999].

Figure 1 illustrates the basic structure of IAMs employed for climate policy analysis. These models aim to represent the causal chain through which (i) economic activities trigger anthropogenic greenhouse gas emissions, (ii) emissions of greenhouse gases translate into atmospheric concentration, temperature shift, and climate change, and (iii) climate change feeds back via the ecosystem to the economy.

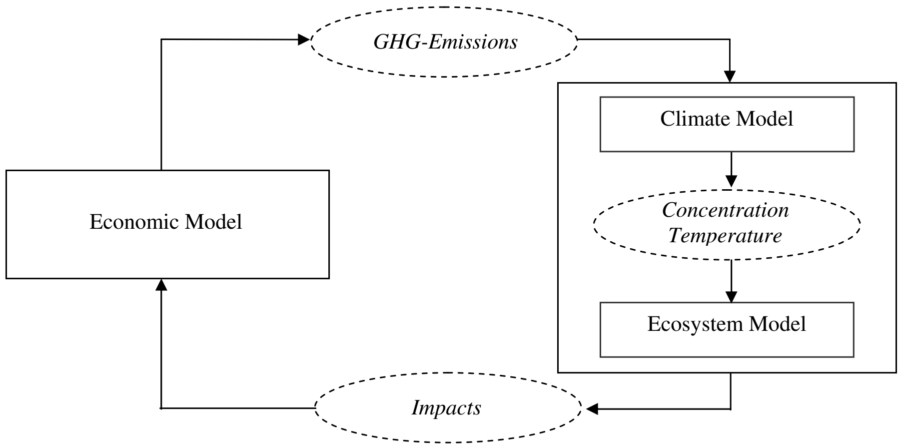

Figure 1: *Schematic Structure of Integrated Assessment Models for Climate Change*

Weyant et al. [1996] distinguish two broad classes of IAMs of climate change: *policy* *simulation models* which assess specific policy measures and *policy optimization models* which seek optimal policies. Policy simulation models typically are used to evaluate the impact of a specific exogenous policy. Avoiding optimization, these models are descriptive and can contain much greater modeling detail on bio-/geophysical, economic or social aspects (see e.g. the *Integrated Model to Assess the Greenhouse Effect – IMAGE* – by Rotmans [1990]). As a downside, the impacts investigated in detailed simulation models may be more difficult to interpret [Kelly and Kolstad 1999].

Policy optimization models are normative in the sense that they seek to derive an "ideal" best-response policy, usually defined from an economic efficiency viewpoint. Assuming rational behavior of economic agents, policy instruments such as emission control rates or emission taxes are derived given explicit objectives, e.g., maximizing social welfare or minimizing the social costs of meeting exogenous environmental targets. Two prominent examples of optimizing IAMs cast as nonlinear programs are the *Dynamic Integrated Climate Economy* (DICE) model by Nordhaus [1994] and *A Model for Evaluating Regional and Global Effects of GHG reduction policies* (MERGE) by Manne and Richels [1992], both of which incorporate stylized representations of the global economy and the global carbon cycle.

From our point of view there are two key difficulties with policy optimization IAMs in the literature. First, integrated assessment models must be solved over very long time horizons in order to provide a consistent accounting of both the costs and benefits of climate policy. The overall model horizon is dictated by the climate component which is typically run over two to three hundred years. When climate and economic equilibria are solved as a simultaneous system, the need to run over a very long horizon demands a sparse level of modeling detail in order to keep the optimization algorithm tractable. For this reason, optimizing IAMs are based on compact representations of both the socioeconomic and natural science systems. A second disadvantage of optimizing IAMs is due to their traditional formulation as nonlinear programs which do not readily admit second-best effects such as preexisting tax distortions.

Thus, "optimal" policies emerging from IAMs formulated as nonlinear programs are only optimal in a perfect, undistorted economy.

We present a new approach for IAMs of climate change which overcomes these two central shortcomings. A decomposition of the economic and climate components allows us to run these sub-models on different time scales. We solve the climate model over a long time horizon in order to produce a precise approximation of climate dynamics and future climate state, and we solve the economic model, formulated either as a nonlinear program or as a mixed complementarity problem (MCP – see Rutherford [1995]), over a shorter time horizon, consistent with the decades in which policy design is relevant. A shorter horizon in the economic model expands the scope for policy-relevant details on other model dimensions such as regional or sectoral disaggregation.[1] Furthermore, our procedure is readily applied to economic models posed as complementarity problems, hence providing the opportunity to incorporate second-best effects. Policy-relevant complexities such as distortionary taxes and market failures (e.g. knowledge spillovers) can then be accounted for in the policy design process.

A third important benefit of our decomposition – independent of the IAM's representation as an optimization problem or a mixed complementarity problem – is the separation of components from different disciplines through a consistent interface as the object of interdisciplinary collaboration.

The remainder of this paper is as follows. In section 2, we lay out the generic decomposition approach and explain how this accomodates a complementarity formulation of the

[1] Chang [1997] uses Benders decomposition approach to the solution of the MERGE integrated assessment model [Manne et al. 1995]. MERGE is thereby decomposed into early and late periods and these two sub-models are solved iteratively to produce intertemporal optimality. Unlike our approach, however, Chang's representation of the MERGE model retains both economic and climate components in an integrated optimization problem whereas our formulation explicitly separates the economic and climate science components which may then operate on different time scales.

economic model. In section 3, we demonstrate the advantages of the decomposition for approximating the infinite horizon of the DICE model, a prototype optimizing IAM in the field of climate change policy analysis. We then extend the basic DICE setting with public goods funded through distortionary taxation in order to illustrate the importance of a second-best setting for the design of climate policies. In section 4, we conclude. An algebraic summary of the alternative DICE formulations is provided in Appendix A. Programming codes for the numerical models are listed in Appendix B which can be downloaded from WWW.MPSGE.ORG.

2 Decomposition

Policy optimization models of climate change typically adopt a cost-benefit perspective in which the marginal costs of controlling greenhouse gas emissions are balanced against the marginal damages induced by those emissions. Climate change impacts are portrayed by a "damage function" which features parametric relationships between economic losses and changes of the climate state. The damage function can be based on explicit models describing climate change impacts in natural vegetation, agricultural yields, water availability, etc. In compact IAMs such as DICE, climate change damages are often related in reduced form to the global mean temperature. Damages may affect either or both consumption and production activities.

In stylized terms we formulate the climate policy problem as a nonlinear optimization problem (NLP) of a representative infinitely-lived agent:

$$\max \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t U(C_t, D_t) \tag{1}$$

$$
\begin{aligned}
\text{s.t.} \qquad C_t &= F(K_t, D_t, E_t) - I_t \\
K_{t+1} &= (1-\delta)K_t + I_t \\
D_t &= D_t(T_t^E) \\
T_t^E &= H(S_t) \\
S_{t+1} &= G(S_t, E_t) \\
K_0 = \bar{K}_0, &\qquad S_0 = \bar{S}_0
\end{aligned}
$$

where:

ρ is the discount rate,

U denotes instantaneous utility reflecting both final consumption and the disutility of climate damages,

C_t represents consumption in period t,

F characterizes aggregate production in period t as a function of capital, damages (with potentially adverse effects on productivity), and emissions,

D_t denotes damages of climate change in period t,

T_t^E is the global mean temperature in period t,

K_t is the capital stock in period t (with $K_0 = \bar{K}_0$ as the initial capital stock),

E_t are emissions in period t,

I_t is investment in period t,

H describes the functional relationship between the climate state and temperature,

S_t is a vector of the climate state (with $S_0 = \bar{S}_0$ as the initial climate state), and

G characterizes the motion of the climate state as a function of the previous climate state and current anthropogenic emissions.

We merge the relationships $T_t^E = H(S_t)$ and $S_{t+1} = G(S_t, E_t)$ into a single equivalent equation

$$T_t^E = \Gamma_t(S_0, E_0, E_1, ..., E_{t-1}), \tag{2}$$

where Γ_t relates temperature in period t as a function of the initial climate state and emissions in previous periods.

Our decomposition is based on a linear approximation of the climate response to anthropogenic activities, i.e. emissions, of the economic system:

$$T_t^E \approx \bar{T}_t^E + \sum_{\tau=0}^{t} \gamma_{t\tau}(E_\tau - \bar{E}_\tau) \tag{3}$$

where

$\bar{T}_t^E$ is the reference value of temperature in period t,

$\bar{E}_\tau$ is the reference emissions in period τ, and

$\gamma_{t\tau}$ denotes the gradient of climate response (temperature) in period t to anthropogenic emissions in period $\tau < t$.

Within the economic model the values of $\gamma_{t\tau}$ are treated as constants. The climate model is nonlinear, so iterative refinement of the Jacobian is required. They are updated in each outer iteration of the decomposition algorithm as:[2]

$$\gamma_{t\tau} = \left. \frac{\partial \Gamma_t(S_0, \vec{E})}{\partial E_\tau} \right|_{\vec{E}=\bar{E}} \tag{4}$$

In our implementation of DICE, the Jacobian for the climate sub-model is approximated with numerical differencing:

$$\gamma_{t\tau} \approx \frac{\bar{T}_t^E - \Gamma_t(S_0, E_0, ..., \bar{E}_\tau + \epsilon, ..., \bar{E}_{t-1})}{\epsilon}. \tag{5}$$

This procedure quickly converges for our illustrative application.

Figure 2 summarizes the basic decomposition approach for our simplified climate policy problem. We start from a reference emission trajectory $\bar{E}$ which is provided by the economic model to the climate model. The climate model calculates the associated point impacts and emissions sensitivities (i.e., $\bar{T}_t$, the temperature trajectory, and $\gamma_{t\tau}$, impacts on temperature

[2] Likewise, the reference values for temperature, $\bar{T}_t^E$, and emissions, $\bar{E}_\tau$ are updated.

in period t of emissions in period τ). These information are returned to the economic model for a subsequent optimization based on a linear approximation of the climate impact. The decomposition process is iterated to convergence. Were there multiple emission sources or greenhouse gases g emitted over time periods t, we would compute one numerical difference for each gas/time period, rendering a total of $|g| \times |t|$ simulations.

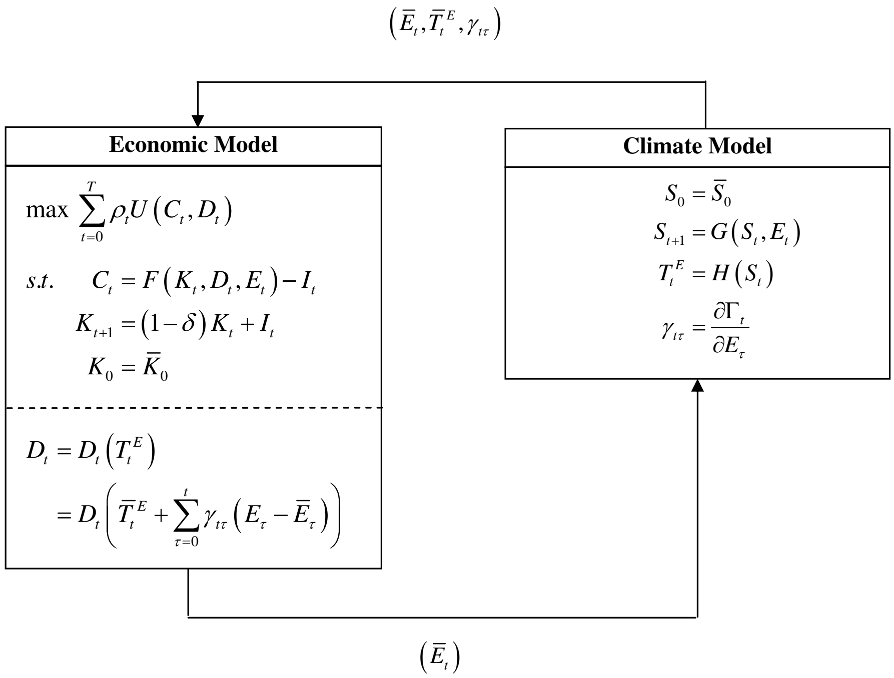

Figure 2: *Basic Decomposition Approach*

Numerical differencing is only computationally tractable for small-scale climate models with solution times measured in seconds. However, our decomposition approach can still be applied to moderate-scale climate models with tolerable solution time (say hours) by using existing techniques for finding the Jacobian of climate variables.[3] Furthermore, for many policy applications and thought experiments, the use of highly simplified reduced-

[3]Sensitivity analyses of climate models is commonly employed to obtain an optimal fit between model results and observations. In order to avoid a computationally expensive approximation of the Jacobian by finite differ-

form climate models seems appropriate. For example, large-scale coupled general circulation models (GCMs) which are the most reliable instruments currently available for the estimation of anthropogenic climate change can be replaced by very compact reduced-form models with solution times in seconds provided they are properly calibrated to the underlying GCM (see Hoos et al. [2001]).

An obvious benefit of our decomposition is that it permits the economic model and the climate model (either in complex or reduced form) to be developed by separate teams with experts from the respective disciplines. Likewise, the approach permits decomposition of effects associated with the different model components – it becomes, for example, easy to interchange the climate model as part of a sensitivity analysis of policy proposals.

A more subtle advantage of the decomposition relates to differences in the nature of time scales for economic and climate models. Intertemporal optimization by economic agents requires that the economic model be solved *simultaneously* over a time horizon sufficiently long to trace the transition toward a new steady-state, [Lau et al. 2002]. Current investment depends on future returns to capital, future emissions, future damages, etc. In contrast, the climate model can be evaluated *recursively* given emission paths from the economic model. Decomposition can therefore simplify the numerical calculation, as it is no longer required to solve the climate model as a simultaneous system of equations.

A further advantage of the decomposition is realized when we formulate the underlying economic model as a mixed complementarity problem (MCP). The MCP framework exploits the complementarity features of economic equilibrium, thereby including the NLP representation of economic equilibrium as a special case (Mathiesen [1985], Rutherford [1995]). By forming the Lagrangian and differentiating, a nonlinear program can be posed as a complementarity problem based on Karush-Kuhn-Tucker conditions. The MCP formulation relaxes

ences, climatologists are developing automated methods to calculate derivatives analytically based on source code of the climate model (see Giering and Kaminski [1998]).

the so-called "integrability constraints" imposed by the NLP framework; one can directly address second-best settings that reflect initial inefficiencies.

Projecting Post-Terminal Emissions and Mitigation

The Ramsey model, which provides the basis for nearly all policy-oriented IAMs, is an "exogenous growth model" (see Barro and Sala-I-Martin [1995], Chapter 2). Primary factor supplies and the intertemporal discount rate are both model inputs, so the long-run growth rate and interest rates are both known. A policy shock in the Ramsey model produces changes in levels but not in growth rates. For this reason, we can estimate emissions paths and damages in the post-terminal period provided that we have an accurate approximation of prices and quantities through the transition period.

Shadow prices on climate impacts are Lagrange multipliers in the NLP and explicit variables in the MCP model. These values provide a means of balancing the near-term cost with the long-term benefits offered through emissions abatement. An economic cost undertaken in period t (the cost of which is reflected in the shadow price of emissions abatement in that year) provides benefits for subsequent periods $\tau > t$ in the same way that capital formation in year t leads to a stream of capital services in subsequent periods.

A linear approximation to the climate model describes the time profile of marginal benefits associated with emission reductions. The first order condition for emissions in year t compares the cost of abatement with the benefits of the reduction in emissions in later periods of the economic model and in those periods which lie in the post-terminal period:

$$-p_t \frac{\partial F}{\partial E_t} = \sum_{\tau=t}^{\infty} \frac{\partial \Gamma_\tau}{\partial E_t} p_\tau^D = \sum_{\tau=t}^{T} \frac{\partial \Gamma_\tau}{\partial E_t} p_\tau^D + \sum_{\tau=T+1}^{\infty} \frac{\partial \Gamma_\tau}{\partial E_t} \tilde{p}_\tau^D \tag{6}$$

where

p_t is the price of the aggregate production good in period t,

p_τ^D is the price (cost) of damage in period τ, and

$\tilde{p}^D_\tau$ is the marginal cost of damage projected in a period $\tau > T$, based on terminal damage and the post-terminal interest rate ($\bar{r}$):

$$\tilde{p}^D_t = p^D_T \left(\frac{1}{1+\bar{r}}\right)^{t-T}.$$

While there are similarities between economic and climate investments, there are substantial differences in the time frame over which these investments pay off, as is illustrated in Figure 3. This figure considers the marginal contribution of benefits over future years of two different types of "investment" in year 80 in the DICE model. The time path labeled "climate" evaluates the discounted return to a marginal reduction in greenhouse gas emissions in year 80 while the path labeled "capital" measures the stream of discounted returns to an additional unit of physical capital formation in year 80. At the margin both types of investment are just profitable, but the time frame over which the benefits accrue is much longer in the case of climate capital than in the case of physical capital. This difference explains in large part why our decomposition procedure works so well. Climate effects operate over a longer time scale than economic effects, and for this reason the climate model needs to operate over a longer horizon than the economic model.

In contrast, conventional IAMs employ "transversality" weights in the objective function which reflect post-terminal climate impacts. The specification of the values for these parameters remains ad-hoc [Nordhaus 1994] and can have substantial impact on results, as we demonstrate below.

Integrability Constraints

First-order conditions of mathematical programs only correspond to equilibrium conditions for the case of integrability that implies efficient allocation (see e.g. Takayma and Judge [1971]).In practical terms, integrability refers to a situation where the shadow prices of programming constraints coincide with market prices. Since many interesting economic problems are associated with non-integrable second-best situations – e.g. due to ad-valorem

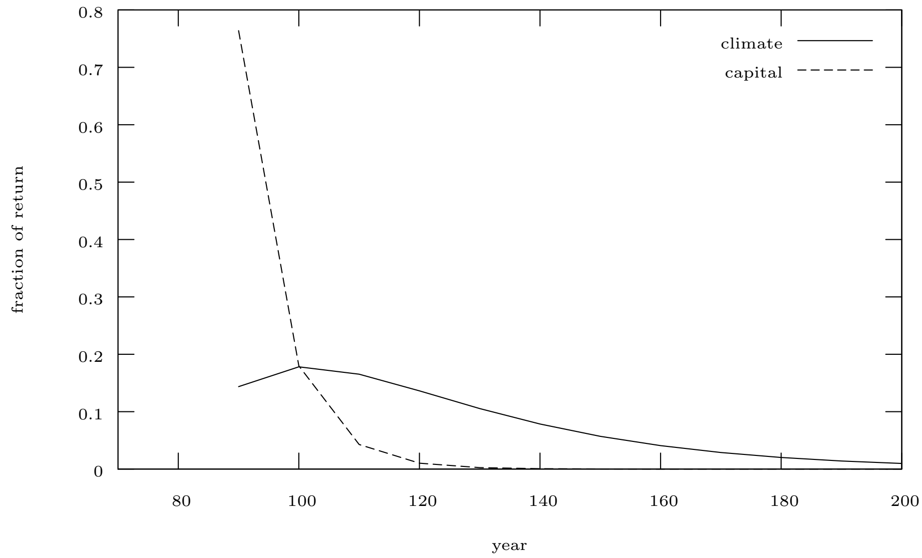

Figure 3: *Time Structure of Returns to Economic and Climate Investments*

taxes, institutional price constraints, or spillover externalities – the classical optimization approach to integrated assessment is relatively limited in the scope of policy applications.[4] In contrary, the MCP formulation of economic problems permits the incorporation of "non-integrabilities" to reflect inefficiencies of market allocation.

3 Illustration

We illustrate the advantages of our decomposition approach using the DICE model by Nordhaus [1994]. This model was originally formulated as a nonlinear program in an integrated, simultaneous system of equations. Because of its simplicity and relative transparency, DICE and its multiregional extension, RICE [Nordhaus and Yang 1996], have been widely used for the integrated assessment of climate change. DICE is based on Ramsey's model of saving

[4]Integrability problems may be relaxed in the optimization context by adding terms to the objective and solving a sequence of nonlinear programs to obtain a market equilibrium (see e.g. Manne and Rutherford [1994]). However, sequential joint maximization with tax distortions is tedious and error-prone.

and investment. A single world producer-consumer chooses between current consumption, investment in productive capital, and costly measures to reduce current emissions and slow climate change. Population growth and technological change (productivity growth) are both exogenous. The representative consumer maximizes the discounted utility of consumption over an infinite horizon subject to a Cobb-Douglas production function which includes damages from climate change as a quadratic function of changes in global mean temperature. In the absence of abatement measures, anthropogenic emissions occur in direct proportion to output. Emissions per unit output are assumed to decline exogenously at a fixed rate and can be further reduced by costly emission-control measures. Within a simple reduced-form "two-box" (ocean and atmosphere) climate sub-model based on Schneider and Thompson [1981], emissions accumulate and increase the stock of greenhouse gases in the atmosphere. As this stock grows, it increases the amount of solar radiation trapped by the earth's atmosphere which in turn triggers an increase in global mean temperature.

For our illustrative application of the decomposition approach, we distinguish two alternative mathematical formulations of DICE: the familiar implementation as an integrated model (INT) and its representation as the combination of separate climate and economic models (DEC). The INT implementation adopts the terminal constraints ("transversality" adjustment terms) as suggested by Nordhaus, where as the DEC implementation employs cost-benefit calculus of climate impacts through the climate model.[5]

3.1 Horizon Sensitivity

In order to evaluate the sensitivity of the optimal policy with respect to the model horizon, we run both models for horizons of 5, 10, 20, and 40 periods (with each period representing

[5]The algebra for both models is provided in Appendix A (provided in this paper) and GAMS code for these models is provided in Appendix B (available from WWW.MPSGE.ORG) . We provide coding of the decomposed model in both NLP and MCP formats, and these formulations produce identical results.

a 10-year time interval). The decomposed model uses an economic horizon of the specified length but runs the climate model over a 600 year horizon. As is evident in Figure 4, the decomposed model is virtually insensitive to the model horizon, whereas the integrated model shows a drastic sensitivity, in particular for the first few decades. The key policy instrument in the DICE model is the emissions control rate, i.e., the fraction of emissions which are mitigated relative to the uncontrolled level. Differences in optimal emission control rates between the two formulations differ substantially, particularly for short time horizons. Precise terminal approximation in the decomposed model offers scope for improvements in the range and details of policy analysis that can be covered, including regional, sectoral or technological details.

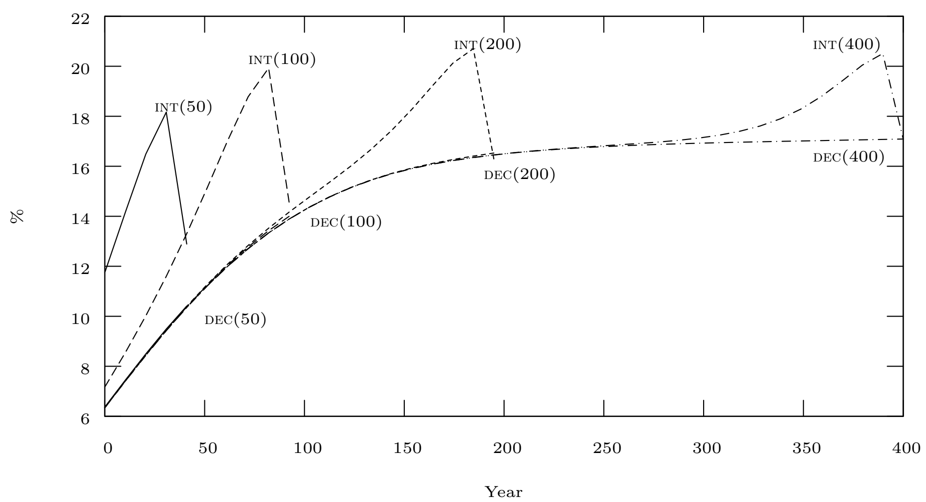

Figure 4: *Sensitivity of Emission Control Rate*

3.2 Revenue Replacement

A decomposed MCP formulation can incorporate second-best effects. We illustrate the importance of market distortions by considering a simple extension of the DICE model in which a public good provided in each period is funded through a distortionary tax on capital earnings. In the reference simulation, we hold the capital tax fixed at an exogenous rate and compute the "optimal" abatement profile together with the resulting level of public goods provision.[6] In the counterfactual simulation we endogenize the capital tax rate through an equal-yield constraint (keeping public good provision at the reference level) and evaluate the marginal utility of deviations from the "optimal" abatement profile for each model period. Carbon taxes then serve two roles in the model. They change relative prices to induce conservation, and they raise public funds thereby providing an opportunity to decrease the capital tax.

As has been well established in the economic literature, preexisting tax distortions affect the economic cost of environmental policy instruments. When the government applies emission restrictions, these raise revenue which may be used to reduce other taxes. In the case where revenues from carbon permit sales are used to replace distortionary taxes, the "optimal" abatement profile based on a first-best setting is too low. This occurs because the marginal benefit calculus in the optimization framework is implicitly based on a marginal cost of public funds equal to 1, whereas distortionary financing of public provision implies that the marginal cost of public funds is greater than one. As illustrated in Figure 5, the larger the baseline tax rate on capital in our example, the larger is the marginal benefit of increasing stringency of environmental restrictions.

4 Conclusions

In this paper, we have presented a new approach to integrated assessment modeling of climate change. Our decomposition of IAMs is based on a linear approximation to the climate model and permits the economic and natural science components to be processed indepen-

[6] "Optimal" – as suggested by the traditional optimization approach – implies that direct marginal abatement cost are equated with the marginal benefits from avoided damages.

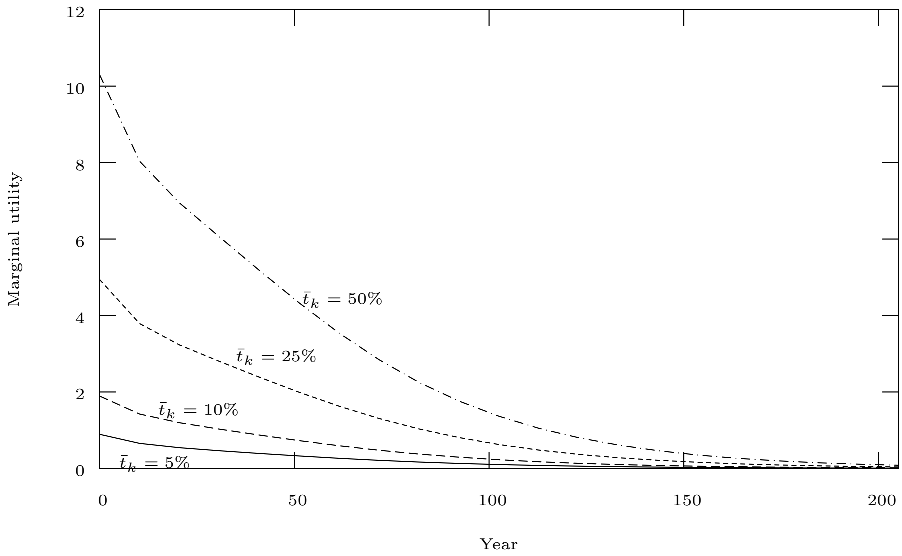

Figure 5: *Welfare Impact of 1% Increase in Abatement*

dently on different time scales. An accurate cost-benefit calculus can be performed with the climate submodel operating over a longer time horizon while the economic model focuses on the policy-relevant near term policy options. From a computational point of view, the reduction in model periods vis--vis integrated models permits more scope for policy-relevant details. Furthermore, a decomposition approach based on a complementarity formulation of the economic system provides a convenient means of incorporating second-best effects that may substantially alter policy conclusions based on the assumptions of perfectly undistorted economies.

Our decomposition allows the separation of components from different disciplines through a consistent interface. The economic model generates emission paths, and the climate model returns climate impacts and their partial derivatives with respect to emissions. Furthermore, the decomposition permits assessment of the relative importance of the various model components. For example, it becomes possible to interchange the climate sub-model and evaluate sensitivity of optimal abatement policies with respect to alternative formulations of natural science relationships. We leave such an investigation for future research.

Finally, it should be noted that our decomposition approach may be attractive for higher-dimensional problems featuring sub-components operating on different time scales. For example, the cost-benefit analysis of R&D expenditures can be based on a decomposed side calculation running over a much longer time horizon than the core economic component.

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A Algebraic Model Formulations

We use the DICE model by Nordhaus [1994] – a standard Ramsey model of savings and investment that combines stylized representations of the global economy and the climate – in order to illustrate the advantages of the decomposed mixed complementarity framework for integrated assessment.

In section A.1, we start with the original implementation of DICE as a nonlinear program (NLP). In section A.2, we proceed with the decomposition of the integrated economy-climate model while maintaining the NLP formulation of the economic sub-model. In section A.3, we re-cast the NLP formulation of the economic sub-model as a mixed complementarity problem (MCP) thereby making use of state-variable targeting for the economic sub-model and cost-benefit calculus through the climate sub-model to better approximate the infinite horizon. In section A.4, we lay out a simple public finance extension to account for pre-existing market distortions within the MCP framework.

A.1 Integrated NLP Formulation

The standard assumptions for the Ramsey model imply that the optimal allocation of resources by a central planner who maximizes the utility of the representative agent is identical to the optimal allocation of resources in an undistorted decentralized economy. The first-order conditions of the associated NLP formulation can thus be interpreted as the outcome of idealized competitive markets.

In the NLP setting, the representative agent explicitly maximizes the discounted value of "utility" from consumption subject to a number of economic and geophysical constraints.

Objective function

The economic objective function in DICE is defined as:

$$\sum_{t=1}^{T} \rho_t L_t \log[C(t)/L(t)] \tag{A-1}$$

where:

C_t is consumption in period t,

L_t is the exogenous labor supply in perid t (population growth), and

ρ_t denotes the discount factor.

Economic constraints

The economic model consists of equations describing technology, abatement options, output markets, emissions, and capital accumlation. Gross economic output is given by a standard Cobb-Douglas function:

$$Q_t = a_t L_t^{1-\gamma} K_t^{\gamma} \tag{A-2}$$

where:

Q_t denotes gross economic output,

a_t represents the level of total factor productivity,

K_t is the capital stock in period t (with $K_0 = \bar{K}_0$ exogenously specified), and

γ is the capital value share (capital elasticity in output).

Abatement options are described by a geometric control cost function:

$$A_t = b_1 \Upsilon_t^{b_2} \tag{A-3}$$

where:

A_t is the abatement level in period t,

Υ_t denotes the emission control rate in period t, and

b_1, b_2 are the exogenous paramters of the abatement cost function.

Total emissions are directly linked to gross output. The emission control rate Υ_t describes the endogenous relationship between emissions and gross output:

$$\Upsilon_t = 1 - \frac{E_t}{\sigma_t Q_t} \tag{A-4}$$

where:

E_t denotes the emissions in period t, and

σ_t is an exogenous efficiency improvement factor which scales down the emission intensity of macro production over time.

Output net of abatement and damage costs (both of which measured as loss in output) equals:

$$Y_t = Q_t - A_t Q_t - D_t Y_t \tag{A-5}$$

where Y_t represents net output in period t, and D_t denotes damages of climate change in period t.

In each period, net economic output is divided between consumption and investment:

$$Q_t = C_t + I_t \tag{A-6}$$

The capital stock is determined by the balance between depreciation and capital investment:

$$K_t = (1 - \delta)K_{t-1} + I_t \tag{A-7}$$

where δ denotes the capital depreciation rate.

Geophysical constraints

The climate sub-model in DICE contains four stylized geophysical relationships that link together the different forces affecting climate change: emission accumulation and transportation (carbon cycle), radiative forcing, and temperature-climate relationships for the atmosphere and lower oceans.

Emission accumulation and transportation is defined as:

$$M_t = 590 + \beta E_t + (1 - \delta_M)(M_{t-1} - 590) \tag{A-8}$$

where:

M_t denotes the atmospheric concentration of CO_2 emission,

β is the marginal atmospheric retention rate, and

δ_M represents the carbon transfer rate to deep ocean.

Radiative forcing is a function of CO_2 emission concentration and other non-CO_2 greenhouse gases:

$$F_t = 4.1 \left(\frac{\log(M_t/590)}{\log(2)} \right) + O_t \tag{A-9}$$

where F_t is radiative forcing (i.e. the increase of surface warming in watts per square meter), and O_t represents other greenhouse gases (most notably CH_4 and N_2O) that are taken as exogenous.

Radiative forcings warm the atmospheric layer, which in turn warms the upper ocean, thereby gradually warming the deep oceans. Due to thermal inertia of different layers there are time lags in climate change. The links between radiative forcing and temperature changes in the atmosphere and the deeper oceans are given as:

$$T_t^E = T_{t-1}^E + c_1[F_{t-1} - c_2 T_{t-1}^E - c_3(T_{t-1}^E - T_{t-1}^L)] \tag{A-10}$$

$$T_t^L = T_{t-1}^L + c_4(T_{t-1}^E - T_{t-1}^L) \tag{A-11}$$

where:

T_t^E is the temperature in the atmosphere,

T_t^L is the temperature in the lower oceans, and

c_1, c_2, c_3, c_4 are geophysical parameters of climate dynamics.

Economic-geophysical linkage constraint

The interface between the economic system sub-model and the climate system sub-model is given by an assumed quadratic relationship between atmospheric temperature and climate change damage:

$$D_t = \upsilon(T_t^E)^2 \tag{A-12}$$

where υ denotes a damage coefficient which is calibrated based on the damage level assumed to be associated with CO_2 doubling.

Terminal constraints

Approximation of an infinite horizon economy within a finite horizon numerical model requires "terminal constraints". For example, in the steady state, gross investment is proportional to the capital stock through the growth rate of the labor force and the capital depreciation rate. A typical terminal constraint for investment might then require sufficient investment to cover growth plus depreciation:

$$I_T = (\chi + \delta_M)K_T \tag{A-13}$$

where χ denotes the growth rate of the labor force.

DICE uses this (integrable) constraint on investment in the terminal period together with an adjustment term in the utility function to account for the "consumption" value of terminal capital stock. In addition, adjustment terms are incorporated to reflect post-terminal damages from emission concentrations and temperature. The adjusted objective function then reads as:

$$\left[\sum_{t=1}^{T} \rho_t L_t \log(C(t)/L(t))\right] + \rho_t(\phi^K K_T + \phi^M M_T + \phi^{T^E} T_T^E) \tag{A-1'}$$

where:

ϕ^K is the (positive) "transversality" coefficient for capital,

ϕ^M is the (negative) "transversality" coefficient for emission concentration, and

ϕ^{T^E} is the (negative) "transversality" coefficient for temperature.

A.2 Decomposed NLP formulation

Our first extension of Nordhaus' model involves decomposition of the integrated economy-climate model based on a linear approximation of the climate model. The decomposition replaces the climate equations in the economic model with a reduced-form linear approximation of climate impacts (temperature):

$$T_t = \Gamma_t \approx \bar{T}_t + \sum_{\tau=1}^{t} \frac{\partial \Gamma_t}{\partial E_\tau}(E_\tau - \bar{E}_\tau) \tag{A-14}$$

where:

$\bar{T}_t$ is the reference level value of temperature (climate impact) in period t,

Γ_t renders the temperagture in period t as a function of the intial climate state and emissions in previous periods

$\bar{E}_\tau$ is the reference level value for emissions in period τ, and

$\frac{\partial \Gamma_t}{\partial E_\tau}$ denotes the gradient of temperature in period t to anthropogenic emissions in period τ.

Local dependence of temperature (climate impacts) in period t on emissions in period τ may be calculated through numerical differencing:

$$\frac{\partial \Gamma_t}{\partial E_\tau} = \frac{\bar{T}_t^E - \Gamma_t}{\epsilon} \tag{A-15}$$

where ϵ is a sufficiently small emission interval for numerical differencing.

Linear approximation of the climate model requires that we account for the local dependence of the transversality terms in the objective function on emissions, and we can calculate the gradient of the transversality terms as:

$$\frac{\partial \Omega_T}{\partial E_\tau} = \frac{(\phi^M M_T + \phi^{T^E} T_T^E) - (\phi^M \bar{M}_T + \phi^{T^E} \bar{T}_T^E)}{\epsilon} \tag{A-16}$$

where $\frac{\partial \Omega_T}{\partial E_\tau}$ denotes the local dependence of the transversality terms in the terminal period on emissions in period τ.

Thus, we obtain the adjusted objective function:

$$\left[\sum_{t=1}^{T} \rho_t L_t \log(C(t)/L(t))\right] + \rho_t \left[\phi^K K^T + \sum_{t=1}^{T} \frac{\partial \Omega_T}{\partial E_t}(E_t - \bar{E}_t)\right] \qquad \text{(A-1")}$$

Altogether, the decomposed model consists of an economic sub-model comprising equations (A-1"), (A-2)-(A-7), (A-13), and (A-14), and the climate sub-model compromising equations (A-8)-(A-12), (A-15), and (A-16). We solve the decomposed model iteratively, by first solving the economic model and then using the resulting emissions profile to evaluate the climate model and its derivatives. Successive solutions converge rapidly as the partial derivatives of temperature with respect to emissions turn out to be very stable.

A.3 Decomposed MCP formulation

Next, we provide the algebraic formulation of the decomposed MCP approach to DICE. Following Mathiesen [1985], the economic model can be characterized by two classes of equilibrium conditions that reflect the first-order conditions of the NLP: (i) zero profit conditions for constant returns activities, and (ii) market clearance conditions for goods and factors. The decision variables are two vectors: (i) activity levels for constant returns production, and (ii) prices for goods (services) and factors. In equilibrium, each of these variables is linked to one inequality condition: (i) an activity level to a zero profit condition, and (ii) a price to a market clearance condition.[7] The primal constraints of the NLP economic model constitute the market-clearance conditions for the MCP whereas the shadow prices (dual variables) of these constraints coincide with market prices. Differentiation of the NLP Langragian with respect to the primal variables (activity levels) renders the zero-profit conditions of the

[7] In a model with multiple agents, we must add an additional class of income balances that relate factor income to expenditure of agents (with associated income variables).

MCP for consumption, capital accumulation, investment, net output, gross output, abatement, emissions, damage, and emission control. We indicate the associated complementary variable to each equilibrium condition using the "perp" operator, "⊥".

- *consumption*:

$$\rho_t L_t / C(t) = p_t^C \quad \perp C_t \tag{A-17}$$

where p_t^C is the price of consumption in period t.

- *capital accumulation*:

$$p_t^K K_t = \gamma p_t^Q Q_t + p_{t+1}^K (1-\delta) K_t \quad \perp K_t \tag{A-18}$$

where p_t^Q denotes the price of gross output in period t, and p_t^K is the price of capital in period t.

- *investment*:

$$p_t^C = p_{t+1}^K \quad \perp I_t \tag{A-19}$$

- *net output*:

$$p_t^Y (1 + D_t) = p_t^C \quad \perp Y_t \tag{A-20}$$

where p_t^Y represents the price of net output in period t

- *gross output*:

$$p_t^Q = p_t^y (1 - A_t) - p_t^E \sigma_t (1 - \Upsilon_t) \quad \perp Q_t \tag{A-21}$$

where p_t^E is the price of emissions.

- *abatement*:

$$p_t^A + p_t^Y Q_t = 0 \quad \perp A_t \tag{A-22}$$

where p_t^A denotes the price of abatement.

- *damage*:

$$p_t^D + p_t^Y Y_t = 0 \quad \perp D_t \tag{A-23}$$

where p_t^D is the price of damage in period t.

- *emissions*:

$$-p_t^E = \sum_{\tau=1}^{T} p_\tau^D \frac{\partial D_t}{\partial T_t^E} \frac{\partial \Gamma_t}{\partial E_\tau} + p_T^D \chi_t \quad \perp E_t \tag{A-24}$$

where χ_t is the (parameterized) post-terminal climate impact of emissions in period t (see below (A-16')).

- *emission control*:

$$-p_t^E \sigma_t Q_t = p_t^A b_1 b_2 \Upsilon_t (b_2 - 1) \quad \perp \Upsilon_t \tag{A-25}$$

Terminal Constraints

In the complementarity formulation, the post-terminal capital stock enters as an endogenous variable. Using *state variable targeting* for this variable, we can relate the growth of investment in the terminal period to the growth rate of capital or any other "stable" quantity variable such as macroeconomic output in the model:

$$I_T / I_{T-1} = Y_T / Y_{T-1} \quad \perp KT \tag{A-26}$$

Furthermore, we need a constraint that defines the price of the post-terminal capital:

$$I_t + K_T(1 - \delta) = KT \quad \perp p_T^K \tag{A-27}$$

where KT represents the post-terminal capital stock.

The complementarity model formulation has explicit price indices representing the cost of abatement and the benefits offered through abatement. A linear approximation to the climate model portrays the time profile of marginal benefits associated with emission reductions at different points in time through the economic model. Thus, we can compare the benefits associated with cutbacks in emissions in the later periods of the model with the benefits of those cutbacks in periods which lie beyond the terminal period of the model.

Post-terminal damages are calculated on the basis of the climate model which is solved for several decades beyond the terminal period of the economic model. Extrapolating present value prices and quantities into the post-terminal period then permits us to relate marginal emission throughout the time horizon of the economic model to damages occurring after the terminal period of the economic model. The valuation of post-terminal damages is based on a geometric extrapolation of post-terminal prices, and post-terminal climate is calculated on the basis of post-terminal emission paths which are extrapolated from the economic model:

$$\chi_t = \left(\sum_{\tau=T}^{TC} \frac{\partial D_t}{\partial T_t^E} \frac{\partial \Gamma_t}{\partial E_\tau} \bar{p}_\tau^D \right) / \bar{p}_T^D \tag{A-16'}$$

where $\bar{p}_\tau^D$ is the reference price of damage in period τ, and TC denotes the extended time horizon of the climate model beyond the terminal period T of the economic model.

The decomposed MCP formulation of DICE combines equations (A-2)-(A-7), (A-13), (A-14), and (A-17)-(A-27) for the economic model and equations (A-8)-(A-12), (A-15), and (A-16') for the climate model.

A.4 Decomposed MCP formulation with Distortionary Public Funding

Our final model version extends the MCP formulation of DICE's economic model with a public sector which finances the provision of a public good model through distortionary taxation of capital earnings. The extended MCP model *cum* decomposition can then be used to illustrate the importance of initial market distortions for the formulation of climate response policies.

The modifications and extensions involve:

- *capital accumulation (zero-profit condition)*:

$$p_t^K K_t = \frac{\gamma p_t^Q Q_t}{1 + t_k} + p_{t+1}^K (1 - \delta) K_t \quad \perp K_t \tag{A-18'}$$

  where t_k denotes the tax rate on capital earnings (as the equal-yield instrument).

- *equal-yield constraint for public good provision*:

$$G = \bar{G} \quad \perp t_k \tag{A-28}$$

where G is the level of public good provision (likewise: government demand), and $\bar{G}$ denotes a fixed target level (index) of public good provision.

- *explicit definition of rents on emissions*:

$$\zeta_t = p_t^E E_t - p_t^Y A_t Q_t \quad \perp \zeta_t \tag{A-29}$$

where ζ_t denotes the rents on emissions in period t.

- *government budget constraint*:

$$G \sum_{t=0}^{T} \frac{p_t^C L_t}{L_0} = t_k \frac{\gamma p_t^Q Q_t}{(1+t_k)+\zeta} \quad \perp G \tag{A-30}$$

The decomposed MCP formulation with distortionary taxation combines equations (A-2)-(A-7), (A-13), (A-14), (A-17), (A-18'), and (A-19)-(A-30) for the economic model and equations (A-8)-(A-12), (A-15), and (A-16') for the climate model.

Appendix B: GAMS Code

B-1 dicedata.gms

```
$title  DICE Data based on the revised version of the model as of August 1993

*     Define default solvers here:

option nlp=conopt;
option mcp=path;

*       http://www.econ.yale.edu/~nordhaus/homepage/dicemodels.htm

$if not set t $set t 40
$if not defined t  SET  t Time periods /1*%t%/
$if not defined tc ALIAS (t,tc);

SETS    tfirst(t)       First period,
        tlast(t)        Last period;

SCALARS
        r               Rate of social time preference per year         /.03/,
        gl0             Growth rate of population per decade            /.223/,
        dlab            Decline rate of population growth per dec       /.195/,
        deltam          Removal rate carbon per decade                  /.0833/,
        ga0             Initial growth rate for technology per decade   /.15/,
        dela            Decline rate of technology per decade           /.11/,
        sig0            CO2-equiv-GWP ratio                             /.519/,
        gsigma          Growth of sigma per decade                      /-.1168/,
        dk              Depreciation rate on capital per year           /.10/,
        gamma           Capital elasticity in output                    /.25/,
        m0              CO2-equiv concent. 1965 billion tons carbon     /677/,
        tl0             Lower stratum temperature (C) 1965              /.10/,
        t0              Atmospheric temperature (C) 1965                /.2/,
        atret           Marginal atmospheric retention rate             /.64/,
        q0              1965 gross world output trillions 1989 US$      /8.519/,
        L0              1965 world population millions                  /3369/,
        k0              1965 value capital billions 1989 US dollars     /16.03/,
        c1              Coefficient for upper level                     /.226/,
        lam             Climate feedback factor                         /1.41/,
        c3              Coefficient trans upper to lower stratum        /.440/,
        c4              Coeff of transfer for lower level               /.02/,
        a0              Initial level of total factor productivity      /.00963/,
        a1              Damage coeff for co2 doubling (fraction GWP)    /.0133/,
        b1              Intercept control cost function                 /.0686/,
        b2              Exponent of control cost function               /2.887/,
        phik            Transversality coef. capital                    /140 /,
        phim            Transversality coef. carbon ($ per ton)         /-9/,
        phite           Transversality coef. temp. (B$ per degree C)    /-7000 /;

PARAMETERS
        L(tc)           Level of population and labor,
```

```
        al(tc)          Level of total factor productivity (TFP),
        sigma(tc)       Emissions-output ratio,
        rr(tc)          Discount factor,
        ga(tc)          Growth rate of TFP from 0 to T,
        forcoth(tc)     Exogenous forcings from other greenhouse gases,
        gl(tc)          Growth rate of labor 0 to T,
        gsig(tc)        Cumulative improvement of energy efficiency;

tfirst(t) = yes$(ord(t) eq 1);
tlast(t)= yes$(ord(t) eq card(t));
gl(tc) = (gl0/dlab)*(1-EXP(-dlab*(ORD(tc)-1)));
L(tc)=L0*EXP(gl(tc))*.9;
ga(tc)= (ga0/dela)*(1-EXP(-dela*(ORD(tc)-1)));
al(tc) =a0*EXP(ga(tc));
gsig(tc) = (gsigma/dela)*(1-EXP(-dela*(ORD(tc)-1)));
sigma(tc)=sig0*EXP(gsig(tc));
rr(tc) = (1+r)**(10*(1-ORD(tc)));
forcoth(tc) = 1.42;
forcoth(tc)$(ORD(tc) lt 15) = .2604+.125*ORD(tc)-.0034*ORD(tc)**2;
```

B-2 dice94.gms

```
$title  DICE version 1994 -- with cosmetic revisions

$include dicedata

VARIABLES
        C(t)            Consumption trillion US dollars
        K(t)            Capital stock trillion US dollars
        I(t)            Investment trillion US dollars
        D(t)            Damage
        A(t)            Abatement cost
        Y(t)            Output net abatement and damage costs
        Q(t)            Gross Output
        E(t)            CO2-equiv emissions billion t
        M(t)            CO2-equiv concentration billion t
        MIU(t)          Emission control rate GHGs
        FORC(t)         Radiative forcing - W per m2
        TE(t)           Temperature - atmosphere C
        TL(t)           Temperature - lower ocean C
        UTILITY         Maximand;

POSITIVE VARIABLES MIU, E, TE, M, Y, C, K, I;

EQUATIONS
        UTIL            Objective function
        YY(t)           Output
        AA(t)           Abatement
        DD(t)           Damage
        QQ(t)           Underlying production function
        CC(t)           Consumption
        KK(t)           Capital balance
```

```
        KC(t)           Terminal condition of K
        EE(t)           Emissions process
        FORCE(t)        Radiative forcing equation
        MM(t)           CO2 distribution equation
        TTE(t)          Temperature-climate equation for atmosphere
        TLE(t)          Temperature-climate equation for lower oceans;

CC(t)..         C(t) =E= Y(t) - I(t);
YY(t)..         Y(t) =E= Q(t) - A(t)*Q(t) - D(t)*Y(t);
AA(t)..         A(t) =E= b1 * MIU(t)**b2;
DD(t)..         D(t) =E= (a1/9)*SQR(TE(t));
QQ(t)..         Q(t) =E= al(t) * L(t)**(1-gamma) * K(t)**gamma;
KK(t)..         K(t) =L= (1-dk)**10 * K(t-1) + 10 * I(t-1) + (k0*0.9)$tfirst(t);
KC(tlast)..     dk * K(tlast) =L= I(tlast);
EE(t)..         E(t) =G= 10 * sigma(t) * (1-MIU(t)) * Q(t);
FORCE(t)..      FORC(t) =E= 4.1*(LOG(M(t)/590)/LOG(2)) + forcoth(t);
MM(t)..         M(t) =E= 590 + atret*E(t) + (1-deltam)*(M(t-1)-590) + m0$tfirst(t);
TTE(t)..        TE(t) =E= TE(t-1)+c1*(FORC(t-1)-lam*TE(t-1)-c3*(TE(t-1)-TL(t-1))) + t0$tfirst(t
TLE(t)..        TL(t) =E= TL(t-1)+c4*(TE(t-1)-TL(t-1)) + tl0$tfirst(t);
UTIL..          UTILITY =E= SUM(t, 10 *rr(t)*L(t)*LOG(C(t)/L(t))/0.55)
                   + SUM(tlast, rr(tlast)*(phik*K(tlast)+phim*M(tlast)+phite*TE(tlast)));

*       Assign a naive starting point which is in the domain of the functions:

C.L(t) = 1; K.L(t) = 1; I.L(t) = 1; Y.L(t) = 1; Q.L(t) = 1; E.L(t) =1;
M.L(t) = 1; MIU.L(t) = 1; FORC.L(t) = 1; TE.L(t) = 1; TL.L(t) = 1;
UTILITY.L = 1;

* Upper and Lower Bounds for economic reasons or stability

MIU.UP(t) = 0.99; MIU.LO(t) = 0.01; K.LO(t) = 1; TE.UP(t) = 20;
M.LO(t) = 600; C.LO(t) = 2;

MODEL CO2 /all/;
SOLVE CO2 maximizing UTILITY using NLP;
```

B-3 nlp.gms

```
$TITLE  DICE version 1994 -- NLP Decomposition

$if not set tc $set tc 80
$if not set t  $set t  40

scalar kterm /0/;

set     tc      /1*%tc%/,
        t(tc)   /1*%t%/;

$include dicedata

alias (t,tp);
```

```
PARAMETER
        dref(tc)        Reference values of damage
        eref(tc)        Reference values of emissions
        pdref(tc)       Reference shadow price on damages
        xi(t)           Post-terminal damages
        grad(tc,t)      Local dependence of D(tp) on E(t);

xi(t) = 0;
dref(t) = 0;
grad(tc,t) = 0;
eref(tc) = 0;

VARIABLES
        C(t)            Consumption trillion US dollars,
        K(t)            Capital stock trillion US dollars,
        I(t)            Investment trillion US dollars,
        Y(t)            Output net abatement and damage costs,
        D(tc)           Damage,
        A(t)            Abatement cost,
        Q(t)            Gross Output,
        E(t)            CO2-equiv emissions billion t,
        MIU(t)          Emission control rate GHGs,

        UTILITY         Maximand;

POSITIVE VARIABLES E, Y, C, K, I, MIU;

EQUATIONS
        UTIL            Objective function
        CC(t)           Consumption
        YY(t)           Output
        AA(t)           Abatement
        QQ(t)           Underlying production function
        KK(t)           Capital balance
        DD(t)           Damage
        EE(t)           Emissions process
        TERMCAP         Terminal capital stock constraint;

CC(t)..         C(t) =E= Y(t) - I(t);
YY(t)..         Y(t) =E= Q(t) - A(t)*Q(t) - D(t)*Y(t);
AA(t)..         A(t) =E= b1 * MIU(t)**b2;
QQ(t)..         Q(t) =E= al(t) * L(t)**(1-gamma) * K(t)**gamma;
KK(t)..         K(t) =L= (1-dk)**10 * K(t-1) + 10 * I(t-1) + (k0*0.9)$tfirst(t);
EE(t)..         E(t) =G= 10 * sigma(t) * (1-MIU(t)) * Q(t);
DD(t)..         D(t) =E= dref(t) + SUM(tp, grad(t,tp)/1e6*(E(tp)-eref(tp)));
TERMCAP..       kterm =e= sum(tlast, (1-dk)**10 * K(tlast) + 10 * I(tlast));
UTIL..          UTILITY =E= SUM(t, 10 *rr(t)*L(t)*LOG(C(t)/L(t))/0.55)
                          - SUM(t, E(t) * xi(t));

*       Assign a starting point which is in the domain of the functions:

K.L(t) = k0*0.9 * L(t)/sum(tfirst,L(tfirst));
```

```
Q.L(t) = al(t) * L(t)**(1-gamma) * K.L(t)**gamma;
I.L(t) = (K.L(t+1) - (1-dk)**10*K.L(t)) / 10;
MIU.l(t) = 0.1;
E.L(t) = 10 * sigma(t) * 0.9 * Q.L(t);
A.L(t) =  b1 * MIU.L(t)**b2;
D.L(t) = 0;
Y.L(t) = Q.L(t)*(1-A.L(t))/(1 + D.L(t));
C.l(t) = Y.L(t) - I.L(t);
UTILITY.L = 1;

kterm = sum(tlast(t), K.L(tlast) * Y.L(t)/Y.L(t-1));

MIU.UP(t) = 0.99;
MIU.LO(t) = 0.01;
K.LO(t) = 0.1;
C.LO(t) = 0.1;

option nlp=conopt;

model CO2 /all/;
solve CO2 maximizing UTILITY using NLP;

eref(t) = E.L(t);
pdref(t) = -DD.M(t);
LOOP((tlast,tc)$(not t(tc)),
  eref(tc)  = eref(tlast)  * (sigma(tc)*L(tc))/(sigma(tlast)*L(tlast));
  pdref(tc) = pdref(tlast) * (L(tc)*rr(tc))   /(L(tlast)*rr(tlast));
);

PARAMETERS
        m(tc)           CO2-equiv concentration billion t,
        forc(tc)        Radiative forcing - W per m2,
        te(tc)          Temperature - atmosphere C,
        tl(tc)          Temperature - lower ocean C,
        deltaE          Difference iterval /0.01/;

m(tfirst) = M0;
te(tfirst) = T0;
tl(tfirst) = TL0;

*       Generate GAMS code for the climate model:

$onecho >climatemodel.gms
$onuni
loop(tc,
        m(tc)    = 590 + atret*eref(tc) + (1-deltam)*(m(tc-1)-590) + m0$tfirst(tc);
        forc(tc) = 4.1*(LOG(m(tc)/590)/LOG(2)) + forcoth(tc);
        te(tc) = te(tc-1)+c1*(forc(tc-1)-lam*te(tc-1)-c3*(te(tc-1)-tl(tc-1))) + t0$tfirst(tc);
        tl(tc) = tl(tc-1)+c4*(te(tc-1)-tl(tc-1)) + tl0$tfirst(tc);
        dref(tc) = (a1/9) * sqr(te(tc)););
$offecho
```

```
*       Generate GAMS code for numerical differencing

$onecho >jacobian.gms
        eref(t) = E.L(t);
        pdref(t) = -DD.M(t);
        LOOP((tlast,tc)$(not t(tc)),
          eref(tc)  = eref(tlast)  * (sigma(tc)*L(tc))/(sigma(tlast)*L(tlast));
          pdref(tc) = pdref(tlast) * (L(tc)*rr(tc))   /(L(tlast)*rr(tlast));
        );
$include climatemodel
        D.L(tc) = dref(tc);
        grad(tc,tp) = 0;
        loop(tp, eref(tp) = eref(tp) + deltaE;
$include climatemodel
          grad(tc,tp) = (dref(tc)-D.L(tc))*1e6 / deltaE;
          eref(tp) = eref(tp) - deltaE;);
        dref(tc) = D.L(tc);
        loop(tlast,
          xi(t) = sum(tc$(not t(tc)), grad(tc,t)/1e6*pdref(tc));
        );
$offecho

$include climatemodel
solve CO2 maximizing UTILITY using NLP;

parameter       itrlog(t,*)     Iteration log -- Emission control rate GHGs;
itrlog(t,"iter0") = MIU.L(t);
set iters /iter1*iter6/;
loop(iters,
$include jacobian
        solve CO2 maximizing UTILITY using NLP;
        itrlog(t,iters) = MIU.L(t);

*       Update terminal capital stock:

        kterm = sum(tlast(t), K.L(tlast) * Y.L(t)/Y.L(t-1));
);
```

B-4 mcp.gms

```
$TITLE  DICE version 1994 -- MCP Decomposition

$if not set tc $set tc 60
$if not set t  $set t  40

set     tc      /1*%tc%/,
        t(tc)   /1*%t%/;

$include dicedata

alias (t,tp); alias (tp,tpp);
```

```
PARAMETER
        teref(tc)       Reference values of temperature
        pdref(tc)       Reference present value of damage
        grad(tc,t)      Local dependence of TE(tp) on E(t),
        eref(tc)        Reference values of emissions
        xi(t)           Post-terminal damage value;

VARIABLES
        C(t)            Consumption trillion US dollars
        K(t)            Capital stock trillion US dollars
        I(t)            Investment trillion US dollars
        Y(t)            Output net abatement and damage costs
        D(tc)           Damage
        A(t)            Abatement cost
        Q(t)            Gross Output
        E(t)            CO2-equiv emissions billion t
        MIU(t)          Emission control rate GHGs

        PY(t)           Output
        PQ(t)           Underlying production function
        PC(t)           Consumption
        PA(t)           Shadow price on abatement cost coefficent
        PD(t)           shadow price on damage coefficent
        PK(t)           Capital balance
        PE(t)           Emissions process
        KT              Terminal Capital stock,
        PKT             Shadow price on terminal capital;

POSITIVE VARIABLES MIU, Y, C, K, I, PE;

EQUATIONS
        YY(t)           Output,
        AA(t)           Abatement,
        DD(t)           Damage (linear climate model),
        QQ(t)           Underlying production function,
        CC(t)           Consumption,
        KK(t)           Capital balance,
        EE(t)           Emissions process,

        EQ_C(t)         Consumption trillion US dollars,
        EQ_K(t)         Capital stock trillion US dollars,
        EQ_I(t)         Investment trillion US dollars,
        EQ_Y(t)         Output net abatement and damage costs,
        EQ_Q(t)         Gross Output,
        EQ_A(t)         Abatement,
        EQ_D(t)         Damage,
        EQ_MIU(t)       Emission control rate GHGs
        EQ_E(t)         CO2-equiv emissions billion t
        EQ_PKT          Equilibrium for terminal capital market,
        EQ_KT           Equilibrium for terminal capital stock;
```

```
CC(t)..         C(t) =E= Y(t) - I(t);
YY(t)..         Y(t) =E= Q(t)*(1-A(t)) - D(t)*Y(t);
AA(t)..         A(t) =E=  b1 * MIU(t)**b2;
QQ(t)..         Q(t) =E= al(t) * L(t)**(1-gamma) * K(t)**gamma;
KK(t)..         K(t) =L= (1-dk)**10 * K(t-1) + 10 * I(t-1) + (k0*0.9)$tfirst(t);
EE(t)..         E(t) =G= 10 * sigma(t) * (1-MIU(t)) * Q(t);
DD(t)..         D(t) =E= (a1/9)*SQR(teref(t) + sum(tp, grad(t,tp)*(E(tp)-eref(tp))));
EQ_C(t)..       10 * rr(t) * L(t) / (0.55*C(t)) =E= PC(t);
EQ_K(t)..       K(t) * PK(t)  =G=
                        gamma * PQ(t) * Q(t) + (PK(t+1)+PKT$tlast(t)) * (1-dk)**10 * K(t);
EQ_I(t)..       PC(t) =E= 10 * (PK(t+1) + PKT$tlast(t));
EQ_Y(t)..       PY(t) * (1+D(t)) =E= PC(t);
EQ_Q(t)..       PQ(t) =E= PY(t)*(1-A(t)) - PE(t)*10*sigma(t)*(1-MIU(t));
EQ_E(t)..       -PE(t) =E= SUM(tp, PD(tp)*grad(tp,t)* 2 *(a1/9) *
                        (teref(tp) + SUM(tpp, grad(t,tpp)*(E(tpp)-eref(tpp)))) )
                        + SUM(tlast, PD(tlast)*xi(t));
EQ_A(t)..       PA(t) + PY(t)*Q(t) =E= 0;
EQ_D(t)..       PD(t) + PY(t)*Y(t) =E= 0;
EQ_MIU(t)..     -PE(t)*10*sigma(t)*Q(t) =E= PA(t) * b1 * b2 * MIU(t)**(b2-1);
EQ_PKT..        SUM(tlast, 10 * I(tlast) + K(tlast) * (1-dk/100)**10) =E= KT;
EQ_KT..         SUM(tlast(t), I(t)/I(t-1) - Y(t)/Y(t-1)) =E= 0;

MODEL DICEMCP /CC.PC, YY.PY, AA.PA, QQ.PQ, KK.PK, EE.PE, DD.PD, EQ_C.C,
     EQ_K.K, EQ_I.I, EQ_Y.Y, EQ_Q.Q, EQ_E.E, EQ_A.A, EQ_D.D, EQ_MIU.MIU,
     EQ_KT.KT, EQ_PKT.PKT /;

PARAMETERS
        m(tc)           CO2-equiv concentration billion t,
        forc(tc)        Radiative forcing - W per m2,
        te(tc)          Temperature - atmosphere C,
        tl(tc)          Temperature - lower ocean C,
        deltaE          Difference iterval /0.01/;

m(tfirst) = M0;
te(tfirst) = T0;
tl(tfirst) = TL0;

parameter       teinit(tc)      Tracking of initial temperature trajectory;

*       Generate GAMS code for the climate model:

$onecho >climatemodel.gms
$onuni
loop(tc,
        m(tc)    = 590 + atret*eref(tc) + (1-deltam)*(m(tc-1)-590) + m0$tfirst(tc);
        forc(tc) = 4.1*(LOG(m(tc)/590)/LOG(2)) + forcoth(tc);
        te(tc) = te(tc-1)+c1*(forc(tc-1)-lam*te(tc-1)-c3*(te(tc-1)-tl(tc-1))) + t0$tfirst(tc);
        tl(tc) = tl(tc-1)+c4*(te(tc-1)-tl(tc-1)) + tl0$tfirst(tc);
        teref(tc) = te(tc););
$offecho

*       Generate GAMS code for numerical differencing
```

```
$onecho >jacobian.gms
        eref(t) = E.L(t);
        pdref(t) = PD.L(t);
        LOOP((tlast,tc)$(not t(tc)),
          eref(tc)  = eref(tlast)  * (sigma(tc)*L(tc))/(sigma(tlast)*L(tlast));
          pdref(tc) = pdref(tlast) * (L(tc)*rr(tc))   /(L(tlast)*rr(tlast));
        );
$include climatemodel
        teinit(tc)  = teref(tc);
        grad(tc,tp) = 0;
        loop(tp, eref(tp) = eref(tp) + deltaE;
$include climatemodel
          grad(tc,tp) = (teref(tc)-teinit(tc)) / deltaE;
          eref(tp)    = eref(tp) - deltaE;);
        teref(tc)     = teinit(tc);
        loop(tlast,
          xi(t) = sum(tc$(not t(tc)), grad(tc,t)*2*(a1/9)*
                (teref(tc) + sum(tpp, grad(tc,tpp)*(E.L(tpp)-eref(tpp))))*pdref(tc))
                / pdref(tlast);
        );

$offecho

K.L(t) = k0*0.9 * L(t)/sum(tfirst,L(tfirst));
Q.L(t) = al(t) * L(t)**(1-gamma) * K.L(t)**gamma;
I.L(t) = (K.L(t+1) - (1-dk)**10*K.L(t)) / 10;
MIU.l(t) = 0.1;
A.L(t) =  b1 * MIU.L(t)**b2;
D.L(t) = 0;
Y.L(t) = Q.L(t)*(1-A.L(t))/(1+D.L(t));
C.l(t) = Y.L(t) - I.L(t);
PC.L(t) = 10 * rr(t) * L(t) / (0.55*C.L(t));
PY.L(t) = PC.L(t) / (1+D.L(t));
PQ.l(t) = PY.l(t);
PK.l(t) = PY.l(t);
PA.l(t) = -PY.L(t)*Q.L(t);
PD.l(t) = -PY.L(t)*Y.L(t);
PE.l(t) = -PA.L(t)*b1*b2*MIU.L(t)**(b2-1)/(10*sigma(t)*Q.L(t));
MIU.UP(t) = 0.99;
MIU.LO(t) = 0.01;
KT.L = sum(tlast, K.L(tlast));
PKT.L = sum(tlast, PK.L(tlast)); PKT.UP = +INF;
E.L(T) = 10 * sigma(t) * (1-MIU.L(t)) * Q.L(t);

set     diagitr Diagonalization iterations /iter0*iter4/;

LOOP(diagitr,

$INCLUDE jacobian

        SOLVE DICEMCP USING MCP;
```

```
);
```

B-5 mcptax.gms

```
$TITLE  DICE version 1994 -- MCP implementation with taxes

$if not set tk0 $set tk0 0.25

scalar  tk0     Baseline capital tax rate /%tk0%/;

scalar  g0      Baseline government /1/;

$if not set tc $set tc 60
$if not set t  $set t  40

set     tc      /1*%tc%/,
        t(tc)   /1*%t%/;

$include dicedata

alias (t,tp); alias (tp,tpp);

PARAMETER
        teref(tc)       Reference values of temperature
        pdref(tc)       Reference present value of damage
        grad(tc,t)      Local dependence of D(tp) on E(t),
        eref(tc)        Reference values of emissions
        xi(t)           Post-terminal damage value;

VARIABLES
        C(tc)           Consumption trillion US dollars
        G               Government demand
        K(t)            Capital stock trillion US dollars
        I(t)            Investment trillion US dollars
        Y(t)            Output net abatement and damage costs
        D(tc)           Damage
        A(t)            Abatement cost
        Q(t)            Gross Output
        E(t)            CO2-equiv emissions billion t
        MIU(t)          Emission control rate GHGs

        PY(t)           Output
        PQ(t)           Underlying production function
        PC(t)           Consumption
        PA(t)           Shadow price on abatement cost coefficent
        PD(t)           shadow price on damage coefficent
        PK(t)           Capital balance
        PE(t)           Emissions process
        TK              Capital tax rate
        KT              Terminal Capital stock,
        PKT             Shadow price on terminal capital
```

```
        RENT(t)         Rents on emission permits;

POSITIVE VARIABLES MIU, Y, C, K, I, PE;

EQUATIONS
        YY(t)           Output,
        AA(t)           Abatement,
        DD(t)           Damage (linear climate model),
        QQ(t)           Underlying production function,
        CC(t)           Consumption,
        KK(t)           Capital balance,
        EE(t)           Emissions process,

        EQ_C(t)         Consumption trillion US dollars,
        EQ_K(t)         Capital stock trillion US dollars,
        EQ_I(t)         Investment trillion US dollars,
        EQ_Y(t)         Output net abatement and damage costs,
        EQ_Q(t)         Gross Output,
        EQ_A(t)         Abatement,
        EQ_D(t)         Damage,
        EQ_MIU(t)       Emission control rate GHGs
        EQ_E(t)         CO2-equiv emissions billion t
        EQ_PKT          Equilibrium for terminal capital market,
        EQ_KT           Equilibrium for terminal capital stock
        EQ_G            Government budget,
        EQ_TK           Capital tax rate
        EQ_RENT(t)      Rental rate;


CC(t)..         C(t) + G * L(t)/L0 =E= Y(t) - I(t);
YY(t)..         Y(t) =E= Q(t)*(1-A(t)) - D(t)*Y(t);
AA(t)..         A(t) =E=  b1 * MIU(t)**b2;
QQ(t)..         Q(t) =E= al(t) * L(t)**(1-gamma) * K(t)**gamma;
KK(t)..         K(t) =L= (1-dk)**10 * K(t-1) + 10 * I(t-1) + (k0*0.9)$tfirst(t);
EE(t)..         E(t) =G= 10 * sigma(t) * (1-MIU(t)) * Q(t);
DD(t)..         D(t) =E= (a1/9)*SQR(teref(t) + sum(tp, grad(t,tp)*(E(tp)-eref(tp))));
EQ_C(t)..       10 * rr(t) * L(t) / (0.55*C(t)) =E= PC(t);
EQ_K(t)..       K(t) * PK(t)  =G=
                  gamma*PQ(t)*Q(t)/(1+TK) + (PK(t+1)+PKT$tlast(t)) * (1-dk)**10 * K(t);
EQ_I(t)..       PC(t) =E= 10 * (PK(t+1) + PKT$tlast(t));
EQ_Y(t)..       PY(t) * (1+D(t)) =E= PC(t);
EQ_Q(t)..       PQ(t) =E= PY(t)*(1-A(t)) - PE(t)*10*sigma(t)*(1-MIU(t));
EQ_E(t)..       -PE(t) =E= SUM(tp, PD(tp)*grad(tp,t)*2*(a1/9) *
                        (teref(tp) + sum(tpp, grad(t,tpp)*(E(tpp)-eref(tpp)))))
                        + SUM(tlast, PD(tlast)*xi(t));
EQ_A(t)..       PA(t) + PY(t)*Q(t) =E= 0;
EQ_D(t)..       PD(t) + PY(t)*Y(t) =E= 0;
EQ_MIU(t)..     -PE(t)*10*sigma(t)*Q(t) =E= PA(t) * b1 * b2 * MIU(t)**(b2-1);
EQ_PKT..        SUM(tlast, 10 * I(tlast) + K(tlast) * (1-dk/100)**10) =E= KT;
EQ_KT..         SUM(tlast(t), I(t)/I(t-1) - Y(t)/Y(t-1)) =E= 0;
EQ_TK..         G =e= g0;
```

```
EQ_G..          G * SUM(t,L(t)/L0*PC(t)) =E=
                TK * sum(t, gamma*PQ(t)*Q(t)/(1+TK)) + SUM(t, RENT(t));
EQ_RENT(t)..    RENT(t) =e= PE(t)*E(t) - PY(t)*A(t)*Q(t);

MODEL DICEMCP /CC.PC, YY.PY, AA.PA, QQ.PQ, KK.PK, EE.PE, DD.PD, EQ_C.C,
     EQ_K.K, EQ_I.I, EQ_Y.Y, EQ_Q.Q, EQ_E.E, EQ_A.A, EQ_D.D, EQ_MIU.MIU,
     EQ_KT.KT, EQ_PKT.PKT, EQ_G.G, EQ_TK.TK, EQ_RENT.RENT /;

PARAMETERS
        m(tc)           CO2-equiv concentration billion t,
        forc(tc)        Radiative forcing - W per m2,
        te(tc)          Temperature - atmosphere C,
        tl(tc)          Temperature - lower ocean C,
        deltaE          Difference iterval /0.01/;

m(tfirst) = M0;
te(tfirst) = T0;
tl(tfirst) = TL0;

parameter       teinit(tc)      Tracking of initial temperature trajectory;

*       Generate GAMS code for the climate model:

$onecho >climatemodel.gms
$onuni
loop(tc,
        m(tc)    = 590 + atret*eref(tc) + (1-deltam)*(m(tc-1)-590) + m0$tfirst(tc);
        forc(tc) = 4.1*(LOG(m(tc)/590)/LOG(2)) + forcoth(tc);
        te(tc) = te(tc-1)+c1*(forc(tc-1)-lam*te(tc-1)-c3*(te(tc-1)-tl(tc-1))) + t0$tfirst(tc);
        tl(tc) = tl(tc-1)+c4*(te(tc-1)-tl(tc-1)) + tl0$tfirst(tc);
        teref(tc) = te(tc););
$offecho

*       Generate GAMS code for numerical differencing

$onecho >jacobian.gms
        eref(t) = E.L(t);
        pdref(t) = PD.L(t);
        LOOP((tlast,tc)$(not t(tc)),
          eref(tc)  = eref(tlast)  * (sigma(tc)*L(tc))/(sigma(tlast)*L(tlast));
          pdref(tc) = pdref(tlast) * (L(tc)*rr(tc))   /(L(tlast)*rr(tlast));
        );
$include climatemodel
        teinit(tc)  = teref(tc);
        grad(tc,tp) = 0;
        loop(tp, eref(tp) = eref(tp) + deltaE;
$include climatemodel
          grad(tc,tp) = (teref(tc)-teinit(tc)) / deltaE;
          eref(tp)    = eref(tp) - deltaE;);
        teref(tc)     = teinit(tc);
        loop(tlast,
          xi(t) = sum(tc$(not t(tc)), grad(tc,t)*2*(a1/9)*(teref(tc)
```

```
          + sum(tpp, grad(tc,tpp)*(E.L(tpp)-eref(tpp))))*pdref(tc)) / pdref(tlast);
        );

$offecho

K.L(t) = k0*0.9 * L(t)/sum(tfirst,L(tfirst));
Q.L(t) = al(t) * L(t)**(1-gamma) * K.L(t)**gamma;
I.L(t) = (K.L(t+1) - (1-dk)**10*K.L(t)) / 10;
MIU.l(t) = 0.1;
A.L(t) =  b1 * MIU.L(t)**b2;
D.L(t) = 0;
Y.L(t) = Q.L(t)*(1-A.L(t))/ (1 + D.L(t));
C.l(t) = Y.L(t) - I.L(t);
PC.L(t) = 10 * rr(t) * L(t) / (0.55*C.L(t));
PY.L(t) = PC.L(t) / (1+D.L(t));
PQ.l(t) = PY.l(t);
PK.l(t) = PY.l(t);
PA.l(t) = -PY.L(t)*Q.L(t);
PD.l(t) = -PY.L(t)*Y.L(t);
PE.l(t) = -PA.L(t)*b1*b2*MIU.L(t)**(b2-1)/(10*sigma(t)*Q.L(t));
MIU.UP(t) = 0.99;
MIU.LO(t) = 0.01;
KT.L = sum(tlast, K.L(tlast));
PKT.L = sum(tlast, PK.L(tlast)); PKT.UP = +INF;
TK.FX = tk0;
E.L(t) = 10 * sigma(t) * 0.9 * Q.L(t);

set     diagitr Diagonalization iterations /iter0*iter4/;

LOOP(diagitr,
$INCLUDE jacobian
        SOLVE DICEMCP USING MCP;
);
```