

A General Equilibrium Model for Tax Policy Analysis in Colombia

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Abstract

The paper documents the development of a pilot general equilibrium model for Colombia based on recent (1996) social accounts. This paper is intended to be a guide for the development, specification, and application of a computable model for the Colombian National Department of Planning. As an illustrative calculation, we use the dataset and model to evaluate the marginal cost of raising additional government revenue from different tax sources.

Contents

1	Introduction	4
2	Social Accounting	5
2.1	SAM Layout	6
2.2	Colombian 1996 SAM	7
2.3	Data Management and Model-Building	10
2.4	Details	10
2.5	Checking Consistency	12
2.6	Forensic Calculations	13
3	A Static General Equilibrium Model	15
3.1	General Overview of GE Modeling	16
3.2	Economic Flows	17
3.3	Symbol Table	18
3.4	Functional Forms	21
3.4.1	Production Functions	21
3.4.2	Consumption, Investment and Government	23
3.4.3	Tax Structure	24
3.5	Other Model Features	25
3.5.1	Steady-State	25
3.5.2	Informal Labor Supply	27
3.6	Harris-Todaro Employment	28
3.6.1	Unemployment	28
3.6.2	Migration	30
3.7	Equilibrium Conditions	30
4	Conducting Economic Policy Analysis	31
4.1	Example: Compute the Marginal Cost of Funds	31
4.2	Short Run vs. Long Run Tax Incidence	33
A	Data Processing	34

A.1	The Excel input file	35
A.2	Reading the SAM into GAMS with XLIMPORT	36
A.3	Economic Accounting and Consistency Checks	38
A.4	Accounting Identities	40

1 Introduction

The analysis of economic policy in a micro-consistent framework demands both theory and data. A common theoretical basis for economic analysis is the Shoven-Whalley (1992) applied general equilibrium framework which is quite flexible and can be applied to a large number of economy-wide issues (commercial policy, tax reform, environmental policy, etc.). The Shoven-Whalley approach is normally based on a multi-sectoral dataset which is provided by an input-output table or a Social Accounting Matrix (SAM). In a textbook exposition, the development of the model and the dataset are conceptually separate activities; in practice, however, these two activities proceed in parallel.

This paper is intended to document the development of a pilot general equilibrium model for Colombia based on a recent (1996) input-output table. To do this, we follow a path starting with the Colombian national accounts, then into GAMS-readable format, through structural assumptions and functional forms, and ending with the final model structure. We then take the model through a typical exercise: we calculate the least-cost source of public funds. Because this is an introductory paper, we do not attempt to model non-standard aspects of indirect taxation such as tax avoidance or corruption¹. For concreteness, we formulate a static model with constant-returns to scale. It is understood that this analysis will ultimately provide a point of departure for subsequent assessments of tax policy options based on more complex formulations.

The paper has the following structure. In Section 2, we consider general features of an IO table and a Social Accounting Matrix. Section A describes the process of importing the Social Accounts from a spreadsheet into GAMS. Section 3 presents the key equations in the `mcfmodel.gms` formulation. We conduct an illustrative calculation of the marginal cost of additional revenue from different tax sources in Section 4.

¹However, we do include an “informal” labor sector which does not pay labor taxes

2 Social Accounting

Payments for goods and services can be represented concisely by using a social accounting framework. Different industries, consumers and government agents sell and purchase goods, then those transactions are recorded and combined to form a Social Accounting Matrix (SAM). The information contained in a SAM is the starting-point for most general equilibrium models. See Pyatt and Round (1985) or Ben King's *What is a SAM*, (1981) for a more detailed introduction to SAM construction and interpretation. A more recent treatment such as Keuning and Ruijter, *Guidelines to the construction of a Social Accounting Matrix*, (1998) may also be useful.

The business of interpreting a particular country's accounting data, and creating a sensible economic framework around a SAM is not so obvious. Often, the social accounts will contain more information than necessary for a straightforward GE model. For example, an industry may purchase and sell the same commodity simultaneously. We know that this probably represents transactions for slightly different commodities within a certain sector, but since we aim to build an overview of the economy, this sort of simultaneous transaction represents a redundancy. In this section we will discuss how to interpret a SAM and build a workable CGE model around this data. The Colombian 1996 social accounts are discussed as a specific example of how to build a static GE model from national accounting data. Some important considerations when doing such an exercise are:

- **The type of policy analysis** What is the purpose of the modeling exercise? If the purpose is tax-analysis, then it makes sense to include as much domestic tax information as possible, while other portions of the data (such as specific trade data) can be treated generically. Conversely, a model to analyze international trade patterns would imply a different data aggregation routine and a different model.
- **The scope of the analysis** Is the analysis regional (e.g., is it one state in the United States), national, or global? The scope of the analysis will help determine where the important data is.
- **Quality of data** Some portions of national accounting are almost completely

fictitious. The financial (capital) accounts, for example, are often notoriously inaccurate. Local experts should use their specific knowledge of the accounts to downplay shady reporting and focus upon what is known to be more accurate.

2.1 SAM Layout

The core of a SAM is the Input-Output table for production. The IO table shows production and use of commodities, distinguished by sector. In this case, the Colombia 1996 SAM contains 13 sectors, but other SAMs may contain 100 sectors or more. Each sector uses outputs from other sectors as intermediate inputs. The IO also includes factors of production such as labor and capital, which are sold by households and purchased by different industries.

Beyond the core IO table are government and household activities. These activities include items like government taxation and provision, household savings and private or public investment. The government often represents a substantial portion of the economy in developing countries, so the transfers between firms, households and institutions are usually included. Colombia is no exception, government activity there accounts for almost 25% of GDP.

Other miscellaneous information may be available and possibly important. In the case of Colombia, the informal labor sector represents a large portion of total labor (compared to other countries). Of course, understanding the scope of informal labor supply and tax avoidance is crucial when investigating how best to support the government budget. The large informal labor market in Colombia means labor taxes are not a promising candidate for collecting tax revenues.

Table 1 shows the layout for a typical (rectangular) SAM². The sub-matrix “A” shows industrial production and use of commodities. This is the core portion of the SAM. Sub-matrix “B” contains final consumption data. This portion of the SAM shows who ended up buying final production of each commodity. Notice that *export* is considered to be a final consumer in this framework. Sub-matrices “C” and “D” list consumption of imports and sales of exports. These imports are either consumed

²A rectangular SAM is another way to represent the national accounts. Instead of using Row/Column notation for sales and purchases as in a square SAM, a rectangular SAM uses *negative* figures to represent inputs and *positive* figures to represent outputs.

for intermediate use or final use. There is often a *BOP* element, which shows the relative value of exports minus imports.

Factors of production and other types of *endowments* are included using submatrices “E” and “F”. Submatrix *E* represents the sales of labor and capital to industry and *F* represents most of the household earnings . Institutional transfers, taxes, trade and transportation markups, and any other transaction is usually listed at the bottom of the SAM using components “G” and “H”. Much of the tricky interpretation relates to the transfers and margins at the bottom of a SAM.

Table 1: A Typical (Rectangular) SAM

	INTERMEDIATE USE by Production Sectors				FINAL USE			
	1	2	...j...	n	Private consum.	Gov't consum.	Invest.	Export
Domestic Production by sector	1 2 : i : n	A			B			
Trade	C				D			
Value added: -labor -capital	E				F			
Transfers -taxes -margins	G				H			

2.2 Colombian 1996 SAM

The SAM for Colombia is shown in Table 2. This is a “square” SAM, because it has an equal number of rows and columns. The row-sums and column-sums should be equal for any consistent square SAM. Industrial production as shown in Table 2 is aggregated in this document for presentation purposes only. The 1996 SAM has detailed sectoral tax information, two types of labor (formal and informal), and

two types of firms and capital (public and private). These features are incorporated into the model. The SAM does not offer household information by income class. This means that the current model should focus upon *efficiency* issues, rather than *distributional* impact. Most of the accounts listed to the right of the ROW account are considered transfers in a static model. A dynamic model can account for these accumulated variables more accurately.

Next we will discuss the changes that were made to the SAM while constructing the static CGE model, `mcfmodel.gms`, and we will construct a rectangular SAM as used in the model.

Table 2: Original Square SAM for Colombia 1996
(Industrial Detail Aggregated)

	Manu- facturing	Servcs	Govnt Services	Formal Labor	Informal Labor	Capital	Other Ind.Tax	VAT	Commerc Margin	Transprt Margin
Manufact.	15418.3	11206.1	1112.6							
Service	8007.9	31629.5	5102.3							
Gov Svcs										
Formal L.	6766.1	19990.9	10610.1							
Informal L.	10723.7	14183.3								
Capital	6551.1	20766.1	2026.4							
Other Ind.Tx.	364.8	877.4	188.8							
VAT	958.7	3227.2								
Tariffs	258.7	842.6								
Comm. Marg.	6133.8	-6133.8								
Trns. Marg.	930.7	-930.7								
Indir.Tax	2385.6	119.9								
Subsidies	-81.9	-47.5								
Direct Tax										
Households				37376.4	24906.3	6350.7				
Government						2742.9	1429.5	4186.9	1100.7	
Pub firms						5782.8				
Priv firms						14468.5				
ROW	5467.1	15525.7		3.1						
Acummulation										
Δ Stocks										
Public Inv										
Private Inv										

Original Square SAM for Colombia 1996 (continued...)

	Indirect Taxes	Subsidies	Direct Taxes	H.Holds	Govnt	Public Firms	Private Firms	ROW	Accum- ulation	Δ Stock	Public Invest	Private Invest
Manufact.				24108.5				10704.8		778.5	5.9	550.3
Service				40940.5				4602.1		-219.8	5025.9	16168.1
Gov Svcs				916.9	18122.5							
Formal L.								12.4				
Informal L.												
Capital												
Other Ind.Tx.												
VAT												
Tariffs												
Comm. Marg.												
Trns. Marg.												
Indir.Tax												
Subsidies												
Direct Tax				1555.5		48.5	4133.1					
Households				115.2	4702.1	604.2	8881.3	4178.8				
Government	2505.5	-128.4	5737.1	5593.1	6647.1	1061.1	2453.8	203.9				
Pub firms				442.5		109.2	713.3	33.6				
Priv firms				7714.7	2177.5	950.6	11098.9	736.1				
ROW				678.5	163.4	2489.6						
Acummulation				5727.7	-3823.7	4143.5	7375.2	3855.4				
Δ Stocks									559.7			
Public Inv					5030.7							
Private Inv									16718.4			

2.3 Data Management and Model-Building

Some adjustments to the 1996 social accounting matrix were required in order to produce a dataset consistent with the static model. This section goes through a few of the interpretations for the Colombian data. We feel that GAMS provides the most consistent environment for data adjustments, hence the first step in the process is to import the SAM from the XLS worksheet into GAMS. We imported the 1996 data into gams using `xlimport`, then began compartmentalizing the accounts for the model.

2.4 Details

The SAM for `mcfmodel.gms` is shown in Table 3. We itemize some aspects of the data adjustments and the interpretations below:

Intermediate Inputs The IO table is copied almost exactly as it appears in the 1996 SAM. One change was a “netting” of production outputs and the own-use of output within an industry. For example, the 118.8 million dollars going from `Other Crops` to itself (cell E9 in the spreadsheet) was subtracted from production.

Indirect Taxes Indirect taxes represent a composite of three items from the original SAM: indirect taxes (21), “other” indirect taxes (26), and subsidies (27).

Labor taxes are shown in the original SAM as a single tax on formal labor supply. This tax is divided up and applied at the production level, so that each producer pays a small share of the total labor tax (as an input tax).

Capital Taxes - treated similarly to labor taxes, each production sector pays their share of the total tax as an input tax.

Trade and Transport Margins The SAM contains “margins,” or markups, between production and consumption. The trade margins represent transportation costs or markups for retail shops. We can see that these margins must be paid by most industries, but they are collected (indicated by a negative number in the SAM) by other sectors. The `transport` industry collects most of the transportation margins, and the `service` industry collects commercial margins. The problem arises because

the margins have both positive and negative entries. The traditional interpretation is that they are “negative” inputs to production. However, production functions are not defined for negative numbers, so these margins had to be accounted-for elsewhere. To solve the problem, a separate margin commodity was created for inputs and for outputs. Positive margins were treated as an input to production for most sectors, negative margins were treated as an additional output for the transport, oil, and service sectors. This portion of the SAM is a good example of how some of the more *mysterious* entries must be interpreted.

Foreign Exchange Colombia is a Small Open Economy (SOE) because Colombia’s international trade activities have a minimal impact on world prices. We record trade using the **px** (price of foreign exchange) commodity. **px** represents Colombia’s exchange rate on the world market. For example, if imports fall relative to exports, the decreased demand for **px** will make foreign goods seem relatively less-expensive. In the Rectangular SAM, the **px** row only shows *net exports*. In the model, imports are combined with their domestically-produced counterparts before final consumption. Exports are explicitly sold in exchange for **px**.

Capital Transfers Items 31 and 32 in the original SAM show capital transfers between public and private companies and other agents in the economy. These transfers are “financial” transactions, which are important, especially in monetary economics. But since we are working with real production and consumption, these accounts are omitted.

Resources Payments Natural resources are an important factor for “extraction” industries, because they are a fixed factor. The inclusion of this fixed factor reflects the fact that extraction industries exhibit decreasing returns to scale, so that a developing country cannot simply extract natural resources indefinitely. We included a *resource payment*, which is considered part of the return to capital, and reflects a certain level of fixed inputs. Country experts should consider the best value-share to choose for each industry. The rectangular SAM shows these estimates for the current static model.

Other Items Accumulation (34) and the Change in Stocks (35) have also been omitted from the model along with the capital transfers.

Table 3: Rectangular Social Accounting Matrix for MCFMODEL

	Manufact Industries	Service Industries	Government Services	Government Agent	Investment	Household Agent	Total:
Manufact	44976.4	-10910.2	-1436.7		-1918.7	-30710.8	0.0
Service	-9120.6	68626.7	-4778.0		-20389.9	-34338.3	0.0
Gov Serv			19039.2	-18122.5		-916.6	0.0
FOREX	4690.9	-10376.3		6344.3		-658.9	0.0
Formal L	-7577.5	-18066.0	-10168.5			35812.0	0.0
Informal L	-11285.8	-13620.5				24906.3	0.0
Privat K	-3728.1	-16818.4				20546.4	0.0
Public K			-2026.3	2026.3			0.0
Resources	-2637.6					2637.6	0.0
VAT	-1571.1	-2615.2		4186.3			0.0
TM	-349.6	-751.0		1100.6			0.0
TL	-329.0	-784.5	-441.5	1555.1			0.0
TK	-1135.1	-2998.9		4134.0			0.0
IndTx	-2758.7	-860.1	-188.1	3806.9			0.0
Trans Marg	-945.2	945.2					0.0
Comerc Marg	-8229.1	8229.1					0.0
Investment				-5030.8	22308.5	-17277.7	0.0
Total:	0.0	0.0	0.0	0.0	0.0	0.0	

2.5 Checking Consistency

Since the original 1996 SAM was consistent, our primary goal after including and adjusting the accounts via the static model is that all of the accounts still balance. In the model, we create an aggregate good, called and *Armington Composite Commodity*, which combines domestic production and imports. This aggregate commodity, A_j , is used as an intermediate input or for final demand. At a minimum, we check supply/demand balance using this commodity. On the left side, A_j is a combination of domestic production, imports and (specific to Colombia) trade and transport margins:

$$A_j = D_j + M_j P_j^M + \sum_m (Mrg_j^D - Mrg_j^S)$$

Now, we know that A_j is supplied for intermediate and final demand, so that the following equation must hold:

$$A_j - \sum_i ID_{ij} - C_j - I_j - G_j = 0.$$

If a SAM is not balanced, the analyst figure out what went wrong, and then decide how to remedy the situation. Another issue is the interpretation of certain taxes and subsidies. For example, value-added tax (VAT) can be interpreted as a tax on labor and capital as factors of production, or it can be interpreted as a consumption tax (since investment is not taxed). The tax system adopted for `mcfmodel.gms` is described in Section 3.4.3. The implication of any assumptions are usually checked by conducting a *sensitivity analysis*, where the results from a policy simulation are tested with and without imposing certain modeling assumptions.

In is section we described how to convert a SAM into a CGE model conceptually. We also describe the specific computational details about how to convert the original SAM data, located in an Excel spreadsheet, into a GAMS dataset and model. These details are listed in Appendix A.

2.6 Forensic Calculations

When assessing a new dataset it is helpful to first develop a sense of the key statistics in the social accounting matrix. Table 4 provides some of these indicators, including the export share of market supply ($E/(E+D)$), the labor share of value-added (L/V), the import share of domestic supply ($M/(M+D)$), formal labor share of wage payments ($L_F/(L_F+L_I)$), and (in the final column) sectoral shares of aggregate GDP.

Value-added and indirect tax rates are computed from the SAM and are round to vary considerably across sectors. Likewise, we use the tariff and import rows from the SAM to compute the benchmark tariff rates. All of these tax rates are shown in Table 5.

Table 4: Echo Print of Base Year Value Shares

	$X/(X + D)\%$	$L/V\%$	$M/(M + D)\%$	$L_F/(L_F + L_I)\%$	$GDP\%$
COF	90	100	0	30	1
CRO	0	95	1	19	5
LVS	2	92	6	19	1
FFH	62	23	0	100	3
OIL	70	66	10	38	2
MIN	37	31	0	68	0
THR	35	99	21	28	6
FOD	5	64	5	45	4
NRI	11	53	22	84	4
NSI	18	65	24	67	3
HTC	16	48	57	91	3
CON	0	56	0	61	7
TRN	8	91	6	65	5
ELE	0	26	0	100	3
COM	10	36	4	98	2
SER	0	66	2	51	35
GOV	0	84	0	100	14

1. $X/(X + D)\%$ is the export value-share in total production.
2. $L/V\%$ is the labor value-share in total value-added.
3. $M/(M + D)\%$ is the import value-share in total consumption.
4. $L_F/(L_F + L_I)\%$ is the formal labor share of total labor.
5. $GDP\%$ is the percentage of total GDP.

Table 5: Percentage Tax Rates from 1996 SAM

	VAT	T_Y	T_M		VAT	T_Y	T_M
COF		1		NSI	25	1	5
CRO			2	HTC	53	1	6
LVS			6	CON		1	
FFH		6		TRN	1	1	
OIL		2	4	ELE		-1	
MIN		1	19	COM	21	1	
THR			3	SER	2	1	
FOD	2	1	6	GOV		1	
NRI	24	14	5				

3 A Static General Equilibrium Model

In this section, we work through the model framework for the basic static model: `mcfmodel.gms`. A typical analysis may require a custom-tailored version of this basic model, but the underlying assumptions and model structure will typically remain intact. Thus, the documentation underlying this model can be recycled for subsequent models derived from the `mcfmodel.gms` general structure.

`mcfmodel` incorporates several key elements of the social accounts, including:

- Two types of labor (formal and informal)
- Five sets of tax instruments:
 1. Value-added taxes, applied to primary factor inputs (vat)
 2. Import tariffs (t_M)
 3. Direct taxes on capital (t_K)
 4. Direct taxes on formal labor (t_F)
 5. Indirect taxes and substitutions on production (t_i)
- Armington differentiation of domestic and foreign goods, include a constant-elasticity of substitution aggregation of imports and domestic goods and a

constant elasticity of transformation between goods produced for domestic and export markets

- Constant investment demand
- Constant elasticity of transformation between labor supplied to the formal and informal labor markets. When the elasticity is set to zero, both types of labor are in fixed supply.

3.1 General Overview of GE Modeling

The static model recreates an Arrow-Debreu (1954) general economic equilibrium model³. Each consumer has an initial endowment of labor, capital, and resources, and a set of preferences resulting in demand functions for each commodity. Market demands are the sum of consumer and intermediate demand. All of the consumers are typically combined to form a “representative agent,” with aggregated demand and total endowments. Commodity market demands depend on all prices and satisfy Walras’s law. That is, at any set of prices, the total value of consumer expenditures equals consumer incomes. Technology is described by constant returns to scale production functions. Producers maximize profits. The zero homogeneity of demand functions and the linear homogeneity of profits in prices (i.e. doubling all prices double money profits) imply that only relative prices are of any significance in such a model. The absolute price level has no impact on the equilibrium outcome.

Equilibrium in this model is characterized by a set of prices and levels of production in each industry such that the market demand equals supply for all commodities. Since producers are assumed to maximize profits, and production exhibits constant returns to scale, this implies that no activity (or cost-minimizing technique for production functions) does any better than break even at the equilibrium prices. Mathiesen (1985) has shown that an Arrow-Debreu model can be formulated and solved as a complementarity problem. Accordingly, three types of equations define an equilibrium: market clearance, zero profit, and income balance.

³For a detailed discussion of general-equilibrium, see Arrow and Hahn (1971), and Shoven and Whalley (1992).

3.2 Economic Flows

The relationship between different sectors and consumers in `mcfmodel` is shown in Figure 1. Taxes are discussed in the next section and therefore, for simplicity, do not appear in this figure.

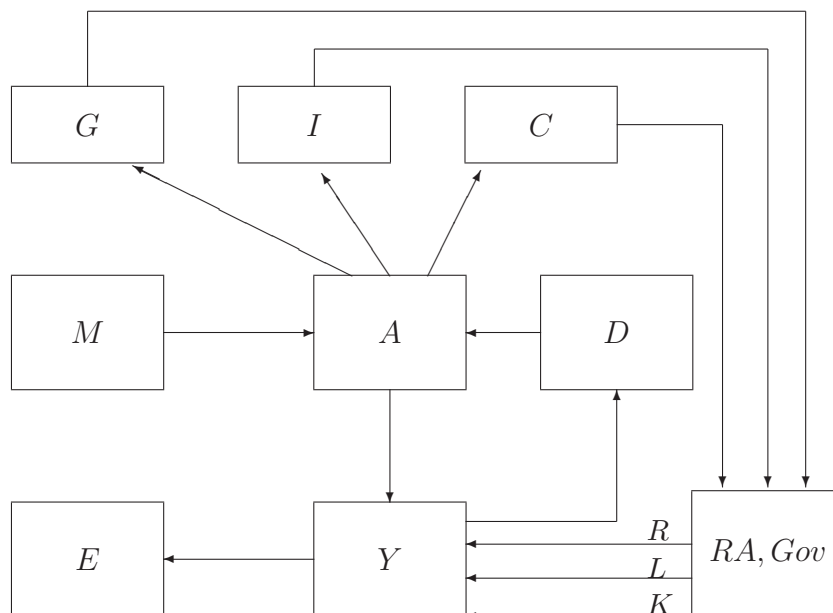


Figure 1: Flows in `mcfmodel.gms`

Production (denoted as Y) combines three factors: capital K , labor L , resources R , and intermediate inputs A , to produce outputs going to the domestic market D or the export market E . An *Armington Composite good* (Armington, 1969) (denoted as A) is a combination of domestic goods D and imports M . Armington aggregate goods are the basic consumption commodity. They are consumed by industry as an intermediate input and they are also goods for final consumption, C , government consumption G , and or investment I . Consumers, RA , and the government, Gov are endowed with factors of production (L, K, R), which get sold to industry (Y). They are also the final consumers, who use income from factor sales to purchase Armington goods (C via A), to invest (I), or to create government services (G).

3.3 Symbol Table

Any model exposition can quickly become confusing when symbols are ambiguous. Key symbols are listed in Table 6.

Note that these symbols may not correspond directly to symbols in the computer code. For example, we use i and j for set identifiers in the paper, but in the `mcfmodel.gms` model code, the set identifiers are `s` and `ss`.

The model has the production sectors detailed in Table 7.

Table 6: Symbol Lookup Table

<u>Set Label</u>	<u>Elements</u>
i (or j)	Sectors (listed in Table 7)
l	Labor types {formal, informal}
k	Capital types {public, private}
m	Margin types {trade, commerce}
<u>Symbol</u>	<u>Description</u>
Y_i	Production of good i
x_{ij}	Intermediate Input: level of A_i used in sector j production
L_i	Formal labor input into sector i
L_{Ii}	<i>In</i> -formal labor input into sector i
K_i	Capital input into sector i
\bar{R}_i	Fixed-supply natural resource input into sector i
A_i	Armington aggregate good (Imports plus Domestic)
E_i	Export output of good i
D_i	Domestic output of good i
M_i	Imports of good i
I_i	Investment demand i
G_i	Government demand
C_i	Household final demand
a_{ij}	Share parameter for factor inputs
<u>Taxes</u>	
t_i, t_F, t_K	Production, Formal-labor, and Capital taxes, respectively
vat_i	Value-added tax
<u>Prices</u>	
p_i	Output price of the Armington aggregate, A_i
w_l	Wage for formal or in-formal labor
rk	Single-period (rental) price of capital
pf_x	Aggregate exchange rate

Table 7: Sectors in the 1996 SAM

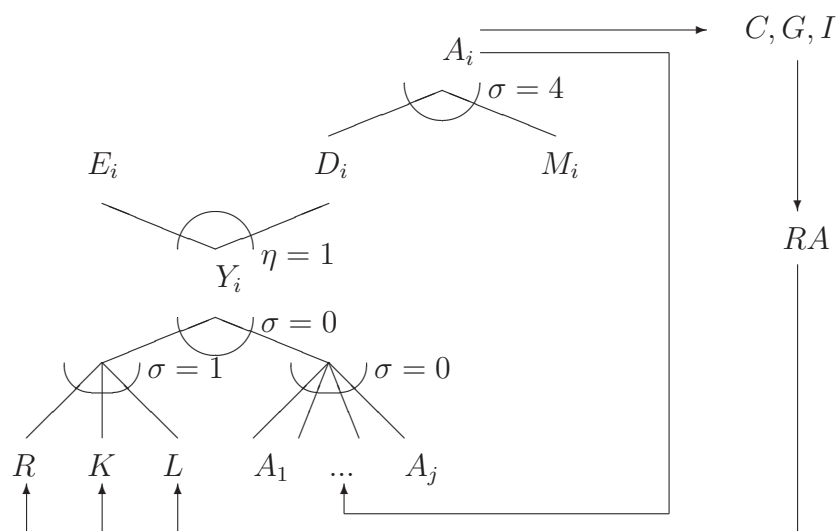
cof	Coffee
cro	Other crops
lvs	Livestock
ffh	Forestry fishing and hunting
oil	Oil
min	Other Minerals
thr	Coffee Threshing
fod	Foodstuffs
nri	Natural Resources Intensive Industries
nsi	Non-skilled Labor Intensive Industries
htc	Capital and High Technology Industries
con	Construction
trn	Transport
ele	Electricity Gas and Water
com	Communications
ser	Private Services
gov	Government Services

3.4 Functional Forms

The Constant Elasticity of Substitution (CES) function is adopted for the static model. CES functions are widely accepted by economists because they are globally regular, and can be defined by their zeroth, first, and second order properties. This means that the location (price and quantity), slope (marginal rate of substitution), and curvature (or convexity) completely characterize a CES production or consumption function. MPSGE is a convenient modeling tool because it accepts these three arguments and automatically constructs a CES function in the model. This allows economists to take a *high-level* approach to production and consumption. Production and consumption structure is defined by showing the linkage between sectors and the elasticity of substitution in consumption and production.

Using this convention, the model structure for `mcfmodel` is shown in Figure 2, where σ is the elasticity of substitution and η is the elasticity of transformation.

Figure 2: `mcfmodel.gms` production structure



3.4.1 Production Functions

Production Inputs Goods are produced according to a nested Leontief-Cobb Douglas technology. Intermediate inputs and aggregate value-added enter at the

top level:

$$Y_i = \min \left[\min_j \left(\frac{x_{ji}}{a_{ji}} \right), \frac{v_i}{b_i} \right]$$

Value-added represents a Cobb-Douglas aggregation of labor, capital and sector-specific resources:⁴

$$v_i = L_{F_i}^{\alpha_F} L_{I_i}^{\alpha_I} K_i^\beta \bar{R}_i^\gamma$$

in which constant returns to scale implies that $\alpha_F + \alpha_I + \beta + \gamma = 1$.⁵

Production Outputs Each production sector Y produces two types of commodities: domestic goods D_i and goods for export E_i . These goods are assumed to be imperfect substitutes, and they have a constant elasticity of transformation. An algebraic formulation of this transformation function is written:

$$Y_i = g(D_i, E_i) = \left[\alpha_i^D D_i^{1+1/\eta} + (1 - \alpha_i^D) E_i^{1+1/\eta} \right]^{1/(1+1/\eta)} \quad (1)$$

where α_i^D is the benchmark value share of domestic sales in total output for sector i and η corresponds to the model input `etrndx`.

Imports The model adopts an Armington representation of the import demand. Armington goods, A_i , are produced by combining domestic goods with imports from the same sector. These goods are treated as imperfect substitutes (e.g., Coffee from Colombia vs. Java). We use σ as the Armington elasticity, which corresponds to `esubdm` in the computer code.

$$A_i = \left(\alpha_i^M M_i^{1-1/\sigma} + (1 - \alpha_i^M) D_i^{1-1/\sigma} \right)^{1/(1-1/\sigma)}$$

⁴The numerical model permits the more general CES functional form for valued-added based on model input `esubk1`. When this input is unity, value-added aggregates are Cobb-Douglas as shown here.

⁵For purposes of illustration we assume that sector-specific resource inputs are a given fraction of the base year capital earnings: Coffee (25), Other crops (25), Livestock (25), Forestry fishing and hunting (25), Oil (75), Other Minerals (75), Natural Resources Intensive Industries (50). Model input `resource` can be used to scale assumed resource shares of base year capital income. When `resource=0`, sector-specific resources are omitted from the model.

Some confusion can arise trying to distinguish between production, Y_i , output (D_i, E_i) and the consumption good (A_i). The Armington aggregate good is the main commodity for use in production and final demand. It combines domestic output, D_i (which is produced via Y_i), with imports, M_i .

Trade Balance The real exchange rate (ρ) is determined by supply of exports and demand for imports, which is determined in units of foreign currency.

$$\sum_i \bar{p}_i^E E_i + B = \sum_i \bar{p}_i^M M_i$$

Holding all else equal, rising import demand will increase ρ , which reflects increased demand for external currency. The fixed parameter B denotes the exogenously-specified current account balance. Because this is a small-open economy, import and export prices (\bar{p}_i^E, \bar{p}_i^M) are fixed exogenously.

3.4.2 Consumption, Investment and Government

Final Consumption A single representative agent (RA) is endowed with primary factors of production: capital, labor, and resources. The RA demands investment, private and government goods. Investment and government output are exogenous, while private demand is determined by utility maximizing behavior. The RA utility function is a Cobb-Douglas:

$$U(A_i) = \prod_i A_i^{\alpha_i} \quad \sum_i \alpha_i = 1$$

The RA maximizes utility subject to a budget constraint:

$$\begin{aligned} & \max_{A_i} U(A_i) \\ & \text{s.t.} \\ & \sum_i p_i A_i \leq p_K K + p_L(L_I + L_F) + p_R R + trn - I \end{aligned}$$

Investment In the static formulation, investment demand is held constant at base-year levels. Investments are aggregated into a single, national investment pool, then distributed among production and government sectors according to base-year

accounts. Investment funds come from households and government. The level of investment can be altered in the steady-state formulation, which is discussed in section 3.5.

Government The government spends money on the purchase of government services and investment. Purchases are supported with tax revenue, capital rents, and net foreign exchange transfers. Total tax revenues are described in section 3.4.3.

3.4.3 Tax Structure

Production inputs are subject to three types of taxes, value-added is taxed at rated vat_i , formal labor is taxed at rate t_F and capital earnings are taxed at rate t_K . Resource inputs are sector-specific, hence their inputs are fixed and the tax applied to resource inputs is lump-sum. Capital and labor allocations are, however, price-responsive. Hence differences in VAT rates across sectors lead to efficiency costs which are captured in the model. Tax-inclusive cost of production is then:

$$Cost_i^Y = \sum_j p_j x_{ji} + (1 + vat_i)[w_F(1 + t_F)L_i^F + w_I L_i^I + (1 + t_K)(rkK_i + r_i \bar{R}_i)]$$

Tax-inclusive output value for Y is:

$$Value_i^Y = (1 + t_i^Y) (p_i^D D_i + \rho \bar{p}_i^X X_i)$$

In equilibrium, the tax-inclusive cost of production equals output value across all sectors, this represents the zero-profit market condition.

Import tariffs are included into the Armington commodity's unit cost function:

$$p_i = \left[\alpha_i^M \left(\frac{p_i^M (1 + t_i^M)}{\bar{p}_i^M} \right)^{1-\sigma} + \left(\frac{p_i^D}{\bar{p}_i^D} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$$

The benchmark tax rate applied on formal labor inputs (t_F) is based on direct tax payment by households in the SAM and gross payments to formal labor. In Colombia, the imputed tax rate on a net basis is 4.3%. The benchmark tax rate applied to private capital (t_K) is based on the direct tax payments by private firms

and the gross payments to capital in all non-government sectors. The imputed tax rate here is 17.8%.

The Colombian static model tax application is shown graphically in Figure 3.

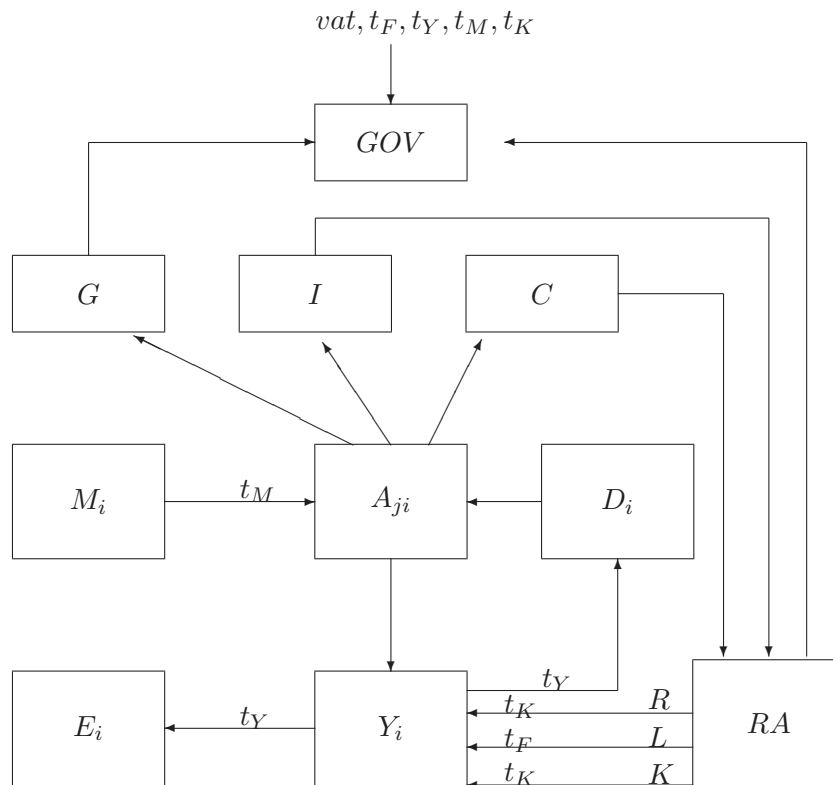


Figure 3: Taxes in mcfmodel.gms

3.5 Other Model Features

3.5.1 Steady-State

A major drawback of static analysis is the presence of a fixed capital stock which does not align with investment. Logically, the level of investment depends upon depreciation, interest rates and the capital stock. Static CGE models usually fail to

address the possible changes to investment and the capital stock the counterfactual. We remedy this drawback by including the *Steady-State* option. The Steady-State feature allows capital and investment to change in response to policy directives, as would happen in a long-run analysis. The adjustment process is consistent with the following complementarity condition:

$$(p_{inv} = rk) \quad \perp \quad \kappa$$

The scale parameter, κ , is complimentary to the steady-state investment equation above, so when rk rises relative to p_{inv} , κ scales up government and private investment to reflect the arbitrage condition. Thus, in the steady-state equilibrium, κ adjusts investment so that investment is consistent with the return to capital. This is done in the MPSGE program using a `$constraint`:

```
$constraint:kstock
      pinv =e= rk("private");
```

`kstock` (κ in the documentation) then scales government and private investment in the `$demand` blocks:

```
$demand:govt
      d:p(s)          q:g0(s)
      e:pinv          q:(-govtinv)    r:kstock
      e:rk(k)         q:govtk(k)     r:kstock
      e:pfx           q:govttrn
```

```
$demand:hh
      d:pc           q:(sum(s,c0(s)))
      e:pinv          q:(-hhinv)     r:kstock
      e:rk(k)         q:hhk(k)      r:kstock
      e:pfx           q:hhtrn
      e:wage          q:(sum(1,ls0(1)))
      e:pr(s)         q:rd0(s)
```

If `kstock` is fixed at unity, then the steady-state feature is disabled, allowing for a short-run comparative-static analysis.

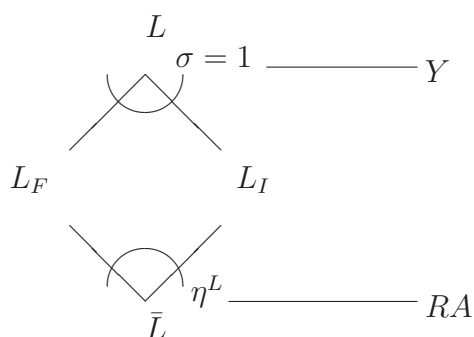
3.5.2 Informal Labor Supply

The labor supply in `mcfmodel.gms` is fixed. However, this labor endowment can be allocated to either formal labor supply (L_F) which is taxed, or *informal* labor (L_I) which is un-taxed. Agents choose how much of each type to supply according to relative wages. The benchmark data reflects a substantial informal labor supply, with almost 40% of the total labor endowment going to the informal sector. The labor-supply unit-revenue function is written:

$$w = \left[\alpha^L \left(\frac{w_F}{\bar{w}_F} \right)^{1+\eta^L} + (1 - \alpha^L) \left(\frac{w_I}{\bar{w}_I} \right)^{1+\eta^L} \right]^{1/(1+\eta^L)}$$

Where η^L represents the model elasticity `etrnl`. The detailed labor supply and demand structure is shown in Figure 4.

Figure 4: Detailed Labor Supply/Demand



The first partial-derivative ($\frac{\partial w}{\partial w_I}$) determines sector-specific labor supply:

$$L_F = \alpha^L \left(\frac{w}{w_F} \right)^{\eta^L}$$

$$L_I = (1 - \alpha^L) \left(\frac{w}{w_I} \right)^{\eta^L}$$

Labor is taxed as an input to production by the direct labor tax (t_F) and value-added taxes (vat). These taxes change equilibrium wages, and the corresponding split between formal and informal labor supply.

3.6 Harris-Todaro Employment

We include a richer description of labor migration and unemployment in the *Harris-Todaro* model, called `htmodel.gms`.

In this formulation unemployment, urban-rural migration, and the real wage are linked. The urban (formal) unemployment rate is determined by a **wage equation**, which uses a wage elasticity parameter, θ . The real wage for formal labor and informal labor is determined by the total labor supply, after migration, and the total demand for each type of labor. Migration between formal and informal labor markets equalize the informal wage and the expected wage in the formal market.

3.6.1 Unemployment

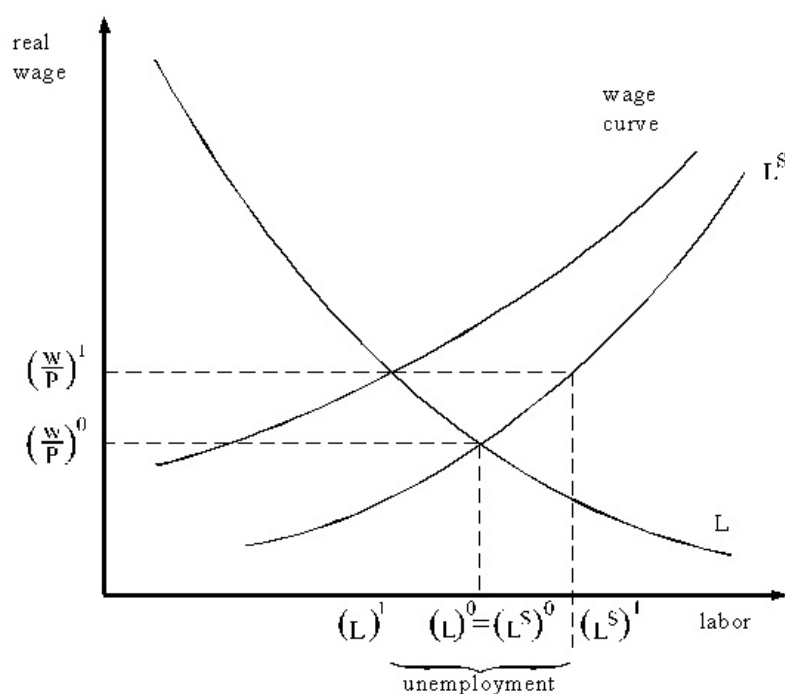
The unemployment rate is determined through a wage equation which postulates a negative relationship between the real wage rate and the rate of employment:

$$\frac{w}{P} = g(ur) \quad (2)$$

where P denotes a consumer goods price index and ur is the *unemployment rate*, taken to be 16% for 1997 in Colombia. This type of wage equation can be derived from trade union wage models, as well as from efficiency wage models (e.g., Hutton and Ruocco, 1999). Figure 5 illustrates the wage curve in a traditional labor market diagram (instead of the $w/p - ur$ space from equation 2). In this figure, the real wage rate is measured on the vertical axis and the quantity of labor is measured on the horizontal axis.

Full employment occurs with the real wage rate of $(w/P)^0$ at the intersection of the (inverse) labor demand function, L , and the formal labor supply function, L^S . Here, we replace the labor supply curve with the real wage curve from equation (2). Consequently, the equilibrium wage rate $(w/P)^1$ lies above the market clearing wage rate. This causes unemployment equal to $(L^S)^1 - L^1$.

Figure 5: The formal-sector wage curve and unemployment



In `htmodel.gms`, we specify the wage equation, $g(ur)$ using an elasticity parameter, θ :

$$\frac{w}{P} = g(ur) = ur^{-1/\theta} \quad \text{if } \theta < \infty \quad (3)$$

and

$$\frac{w}{P} = 1 \quad \text{if } \theta = \infty$$

As $\theta \rightarrow \infty$, the real wage curve approaches a neoclassical, downward-rigid real wage.

3.6.2 Migration

Following Todaro (1970), we link the labor migration rate, the real-wage differential, and unemployment. Migration occurs when the expected real wage stream for

urban employment is high relative to rural (informal) employment. In our treatment, workers migrate into the formal labor sector until informal wages are equal to *expected* formal wages.

$$w^I = (1 - ur) \cdot w^F \quad (4)$$

The expected wage in the formal sector is the wage, w^F , times the employment rate $(1 - ur)$. In the `htmodel`, we specify the initial unemployment rate to be 16%, which was the 1997 rate in Colombia. As ur rises, the gap between formal and informal wages widens.

Labor supply for the formal and informal sectors is determined by the migration rate and the unemployment rate. First, the supply of formal labor is equal to the employed fraction of the workers who chose to migrate to the formal sector:

$$L^F = L_0^F \cdot \frac{1 - ur}{1 - ur^0} \cdot \frac{m}{m^0} \quad (5)$$

where m is the *migration rate* between the informal and formal labor sectors, ur^0 is the initial unemployment rate, and L_0^F is the benchmark formal labor supply. Then the informal labor supply is equal to those workers who did not migrate:

$$L^I = L_0^I \cdot \frac{1 - m}{1 - m^0}. \quad (6)$$

The analyst is free to choose the elasticity of transformation between the formal and informal labor sectors. In the *Overview of the Harris-Todaro model for Colombia*, this elasticity is estimated at $\frac{3}{10}$. In the `htmodel` framework, the net migration level will depend upon this elasticity of transformation, as well as the wage equation parameter, θ , and the unemployment rate.

3.7 Equilibrium Conditions

Three equation classes define an Arrow-Debreu equilibrium in `mcfmodel.gms`:

$$\text{Zero Profits:} \quad \text{Cost}_i(\mathbf{p}) \geq \text{Rev}_i(\mathbf{p}) \quad \perp Y_i$$

$$\text{Market Clearance:} \quad D_i + M_i \geq \sum_j A_{ij} + E_i + RA_i + GOV_i \quad \perp p_i$$

$$\text{Income Balance:} \quad \sum_i p_i A_i \geq w \bar{L} + p_K \bar{K} + p_R \bar{R} + trn - I \quad \text{for (GOV,RA)}$$

Zero Profit The first class of constraint requires that in equilibrium no producer earns an "excess" profit, i.e. the value of inputs per unit activity must be equal to or greater than the value of outputs.

The corresponding complementary variable for a zero profit condition is output (Y_i). Holding all else equal, if output prices rise for commodity i , production activity increases until marginal cost equals marginal revenue.

Market Clearance The second class of equilibrium conditions is that at equilibrium prices and activity levels, the supply of any commodity must balance or exceed excess demand by consumers and producers. The equation above refers to produced commodities, a similar constraint holds for endowed goods like labor, capital and resources.

The corresponding complementary (dual) variable for the market clearance condition is price (p_i or p_F, p_K, P_R, w). Prices adjust until supply equals demand for a given commodity or factor.

Income Balance The third condition is that at an equilibrium, the value of each agent's income must equal the total value of expenditures. We always work with utility functions which exhibit non-satiation, so Walras' law always holds.

4 Conducting Economic Policy Analysis

4.1 Example: Compute the Marginal Cost of Funds

Having implemented the model we do some initial calculations in which we assess the welfare cost of the five tax instruments. In each calculation we proportionally increase tax rates by 10%. The tax streams we evaluate are the value-added tax (revenue 4.2), the import tariff (1.1), the labor tax (1.6), the direct tax on capital (4.1) and other indirect taxes (3.8). When we scale tax rates, consumers and producers adjust behavior to produce a new equilibrium consistent with a new level of government income and expenditure. Government expenditure increases less than proportionally to the tax rate as a result of changes in individual behavior.

Table 8: Marginal Efficiency and Incidence of Base Year Taxes

	<i>YIELD</i>	<i>MCF</i>	<i>MCF_F</i>	<i>MCF_I</i>	<i>MCF_K</i>
VAT	70%	1.27	-0.12	-0.19	-0.71
TY	75%	1.30	-0.02	-0.52	-0.38
TM	66%	1.09	0.11	-0.61	-0.26
TL	71%	1.10	-0.36	-0.18	-0.15
TK	62%	1.10	0.33	-0.18	-1.20

Table 8 indicates the results of calculations with the static model. There is one row for each of the tax instruments. The first column of Table 8 indicates the responsiveness of the tax based to changes in the tax rate. A 70% yield means that when the tax rate is increased by 10%, aggregate tax revenues only increase by 7%. The column titled *MCF* indicates the marginal cost of funds, based on the welfare cost of a marginal tax increase. This column suggests that in a static model the system of indirect taxes (*TY*) is the most costly source of tax revenue while the import tariff (*TM*) is the least costly revenue source, in which the economic cost of raising \$1 of additional public revenue costs roughly \$1.10.

The final three columns in Table 8 indicate the marginal incidence of each tax instrument for formal labor (*MCF_F*), informal labor (*MCF_I*) and capital (*MCF_K*). The marginal incidence indicates the percentage change in the real return to each of these factors per percentage increase in tax revenue. The value of -0.12 for VAT for formal labor indicates that a one percent increase in tax revenue financed through an increase in the VAT produces a 0.2% decrease in the real wage of formal sector workers and a 0.7% decrease in the real return to capital.

4.2 Short Run vs. Long Run Tax Incidence

The cost of additional funds, when taken from a long-run perspective shows us two things. First, the cost of funds is much higher when agents are allowed more time to adjust. The marginal cost of funds (*MCF*) column in Table 9 is about 50% higher when taken from a long-run perspective. This is intuitive, since in

Table 9: Tax Efficiency in the Steady-State

	<i>YIELD</i>	<i>MCF</i>	<i>MCF_F</i>	<i>MCF_L</i>	<i>MCF_K</i>	ΔK
VAT	67%	1.72	-0.45	-0.44	-0.07	-1.65%
TY	73%	1.56	-0.20	-0.71	0.02	-0.88%
TM	64%	1.32	-0.06	-0.79	0.16	-0.21%
TL	70%	1.20	-0.43	-0.24	0.01	-0.14%
TK	57%	1.86	-0.20	-0.65	-0.07	-2.31%

1. The “ ΔK ” column shows the percentage change in the national capital stock.

the long-run, the demand for all goods is relatively more elastic, which implies a less-efficient tax instrument. Second, we see that raising direct labor taxes is less costly in the long-run. This is obvious given that we are now allowing capital stock to adjust to changes in the economy. In most dynamic tax analyses (cite some papers here), investigators find that labor taxation is preferred from an efficiency standpoint because long-run labor supply is relatively inelastic when compared to long-run capital supply. Import tariffs remain a relatively in-expensive source of government revenues relative to capital or indirect taxation.

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A Data Processing

The first step in dealing with a SAM is to transfer the data into GAMS readable format. In order to transfer an Excel file into GAMS format, we use the following:

`sam1996.xls` (Original data file)

`sam.gms` (Data extraction program)

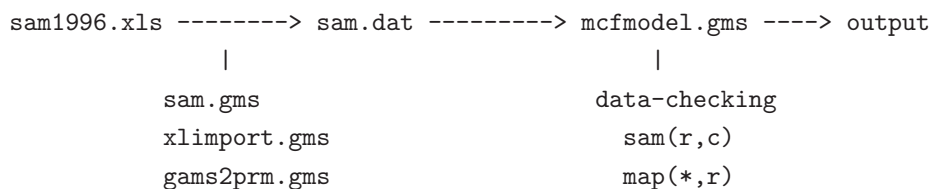
`xllink.exe` (XL conversion utility)

`gams2prm.gms` (GAMS data utility)

If you do not have `xllink.exe` and `gams2prm.gms`, read about how to download and install them at: <http://debreu.colorado.edu/inclib/tools.htm>

We start with the Microsoft Excel file named `sam1996.xls`. The spreadsheet consists of row and column headings, and a 37x37 data matrix. The data flows from the `xls` file into the model as depicted in Figure 6:

Figure 6: Data Flow from Excel to GAMS



First, `sam.gms` manages the data transfer from `.xls` to `.dat`, then `sam.dat` is included directly into the main GAMS model, `mcfmodel.gms`. The first part of `mcfmodel.gms` interprets the national accounts data and checks for consistency. At this point, the data is ready to be included in the CGE model. In order to minimize the number of potential mistakes, we stress the necessity of multiple checks during the process of data transformation and model building.

Converting the data from XL format into GAMS-readable format is a one-time affair. Once the data is converted, only the GAMS dataset (`sam.dat`) is required⁶. Each step in this process is described below.

A.1 The Excel input file

Each account is referenced in the `xls` file using a long description, such as “*Forestry, Fishing and Hunting*”. To facilitate moving the data out of the spreadsheet, row and column index numbers are used to identify each element. The descriptive sector names will be re-applied downstream, in the model itself.

⁶Of course, hold onto the original data in case `sam.dat` becomes unreadable or is deleted!

Figure 7: The 1996 Colombia SAM with Numbered Rows and Columns

	A	B	C	D	E	F
6				Coffee	Other crops	Livestock
7				1	2	3
8	1	Coffee	1		2	1
9	3	Other crops	2	10	119	33
10	4	Livestock	3	5	125	471
11	5	Forestry, fishing and hunting	4	3	27	14
12	6	Oil	5	-	-	-
13	7	Other Minerals	6	-	0	0
14	2	Coffee Threshing	7	-	0	0
15	8	Foodstuffs	8	36	334	648
16	9	Natural Resources Intensive Industries	9	10	57	44
17	10	Non-skilled Labor Intensive Industries	10	22	53	26
18	11	Capital and High Technology Industries	11	169	614	281

A.2 Reading the SAM into GAMS with XLIMPORT

A small GAMS program, called `sam.gms` moves the 1996 data from `xls` format into a GAMS dataset⁷. `sam.gms` uses the spreadsheet import routine, `xlimport`⁸, which must be installed before data can be moved between GAMS and Excel. The basic syntax for `xlimport` is as follows:

```
$LIBINCLUDE xlimport parameter file range
```

where

`parameter` is a name of the GAMS parameter to which data will be retrieved;

`file` is a name of a file from which data will be read;

`range` is a range of data in the `file` which needs to be imported.

This command is part of `sam.gms`, shown below:

⁷It is useful to note that a GAMS dataset is simply a text file which complies with GAMS syntax. GAMS datasets are not binary, and they can be opened and viewed with any text editor.

⁸`xlimport`, `xlexport`, and `xldump` are all functions included in the `xllink.exe` utility. For installation and syntax information, visit <http://debreu.colorado.edu/xllink/xllink.htm>

```

sam.gms (partial listing):
1      set      r /1*37/;  alias (r,c);
2      parameter sam(r,c)  Original SAM data for 1996;
3      $libinclude xliimport sam sam1996.xls samdata
4
5      file kdat/sam.dat/; put kdat;
6      $libinclude gams2prm sam

```

Line 1 defines the row and column dimensions for the spreadsheet to import. Thus, rows and columns 1 ... 37 should correspond to rows/columns in the spreadsheet. Next, the parameter `sam(r,c)` is defined as a 37x37 matrix to hold the original spreadsheet data. `sam(r,c)` is used as a temporary place-holder for the data until economic parameters are defined in the main GAMS model. Line 3 executes the `xllink.exe` utility and extracts the data from the Excel spreadsheet.

A new datafile is created using lines 5 and 6. First an output *handle* (`kdat`) is created. This handle is an alias for the physical file, called `sam.dat`, the GAMS data file. The `gams2prm` utility is used to export `sam(r,c)` into the file `sam.dat`.

Here is the dataset created using `sam.gms`:

```

sam.dat (partial listing):

parameter sam Original SAM data for 1996/
*=>gams2prm sam
* Called from J:\SAM.GMS, line 1479
* 11/29/01 09:41:04
1.2    2.1250985919499E+00
1.3    5.6248830334883E-01
1.4    1.0322435297989E-01
1.6    4.4898020242278E-05
1.7    2.0578320648199E+02
1.8    2.2765243456326E+01
1.9    5.2251918488353E+00
1.10   7.3400583958944E-06
1.11   3.1007968762837E-01
1.12   2.7406671636713E-01
1.14   3.4119510427834E-02
...

```

`sam.dat` is a text file which can be inserted directly into any GAMS program.

The `gams2prm` utility defined the parameter `sam`, using the same description as in `sam.gms`, then wrote out each element of the matrix according to index number. For example, element 1.9, equal to about 5.22 million 1996 dollars, represents the input of *Coffee* into the *Natural Resource Intensive Industries* sector. We insert `sam.dat` into the main GAMS program, `mcfmodel.gms`, then define each data element for use in the model.

A.3 Economic Accounting and Consistency Checks

At this point, we can include the data into `mcfmodel.gms` and make some economic interpretations. The `sam.dat` data file is inserted by using the `$include` directive, as in

```
mcfmodel.gms (partial listing):
$include    sam.dat
```

This pastes the contents of `sam.dat` into the program exactly where the `$include` statement is used.

Data Mapping GAMS provides users with the luxury of using *human* notation for set elements. For example, the `Oil` sector in the model could have been called sector “5”, but a much better abbreviation is something like “oil.” So we define a set of production sectors, `S`, with each row of the SAM as elements:

```
mcfmodel.gms (partial listing):

set      s          Sectors /

cof      Coffee
cro      Other crops
lvs      Livestock
ffh      Forestry fishing and hunting
```

Next, we make an association between the set `S`, and the rows/columns in the dataset. This can be done efficiently by using a temporary set called `map(*,r)`. Where the basis of this set can be anything in the first dimension (denoted by the wildcard symbol, “*”), but only elements of the set `r` in the second dimension

(which contains the digits 1 . . . 37). It is easy to see that the definition of `map` simply connects each element of `s` to the corresponding row in the SAM. Coffee (`cof`) is connected with row number 1, and so on:

```
set map(*,r)    Mapping onto the SAM rows /

cof.1    Coffee
cro.2    Other crops
lvs.3    Livestock
ffh.4    Forestry fishing and hunting
oil.5    Oil
```

Parameter Assignments The base-year economic flows are defined by picking elements from `sam(r,c)`. For example, intermediate inputs are inserted into the parameter `id0(s,ss)` by picking out the diagonal elements:

```
loop((s,ss,r,c)$ (map(s,r)*map(ss,c)),
      id0(s,ss) = sam(r,c);
      sam(r,c) = 0;
);
```

Final consumption is assigned to a parameter called `c0(s)`, and is defined by picking up the elements of column number 29:

```
c0(s) = sam(c,"29");
```

To verify that we are getting the correct element from the 1996 spreadsheet, take a look at the IO table and verify that households consume 12,613 million dollars worth of *Foodstuffs*, then check corresponding values for `c0`:

```
---- 675 PARAMETER C0 Household consumption demand
```

```
thr 3.443,   fod 12.613,   nri 6.552,   nsi 6.602,   htc 6.243
ser 20.924,   gov 0.917
```

As expected, the “fod” listing above shows 12.613 billion, or 12,613 million⁹.

The rest of these parameters are assigned similarly. The `loop` statement repeats the exercise for each sector and column, so long as `map(s,c)` exists. A portion of these assignments is below:

```
loop((s,c)$map(s,c),
*      Extract components of final demand:
      c0(s) = sam(c,"29"); sam(c,"29") = 0;
      g0(s) = sam(c,"30"); sam(c,"30") = 0;
      x0(s) = sam(c,"33"); sam(c,"33") = 0;

*      Extract margin supply and demand:
      md0("trade",s) = max(0, sam("25",c));
      ms0("trade",s) = max(0, -sam("25",c));
      ... and so on ...
```

A.4 Accounting Identities

Some simple accounting checks often help ensure the national accounts have been correctly inserted. For example, we check a consumption-production identity. Domestic consumption, $a_0(s)$, can be calculated two ways, via domestic production and imports:

$$a_s^0 = d_s^0 + m_s^0 \cdot pm_s^0 + \sum_m (md_{m,s}^0 - ms_{m,s}^0)$$

or via final demand, investment, and government:

$$a_s^0 = \sum_{ss} id_{s,ss}^0 + c_s^0 + i_s^0 + g_s^0$$

The equivalence is checked in `mcfmodel.gms` using parameter definitions:

```
a0(s) = d0(s) + m0(s)*pm0(s) + sum(m, md0(m,s)-ms0(m,s));

parameter mktchk(s)      Cross check of consistency;
mktchk(s) = a0(s) - sum(ss, id0(s,ss)) - c0(s) - i0(s) - g0(s);
display mktchk;
```

⁹The values from the original IO table were scaled by 1000, making the unit of measurement, billions of US dollars.

`mktchk` is displayed in the listing file:

```
----      723 PARAMETER MKTCHK          Cross check of consistency
cof 5.80092E-15,   cro 7.21645E-16,   lvs 8.64846E-15,   ffh 2.17465E-14,
oil -7.5530E-15,  min 4.87175E-15,   thr 5.79536E-14,   fod 3.98848E-13,
nri -7.1831E-14,  nsi 7.27196E-14,   htc 1.84741E-13,   con -3.7303E-14,
trn 1.77636E-15,  ele -9.3259E-15,   com 2.84217E-14,   ser 9.85656E-13,
gov 1.70530E-13
```

The accounts are consistent because `mktchk` is a very small number¹⁰.

¹⁰Typically, we consider numbers less than $1e-6$ to be fairly small, and $1e-10$ small enough to be a result of computer tolerances.