# Decomposing Integrated Assessment of Climate Change

*Christoph Böhringer [a,b], Andreas Löschel [a], and Thomas F. Rutherford [c]*

[a] Centre for European Economic Research (ZEW), Mannheim
[b] Faculty of Economics and Social Studies, University of Heidelberg
[c] Department of Economics, University of Colorado

**Abstract** We present a decomposition approach for integrated assessment modeling of climate policy based on a linear approximation of the climate system. Our objective is to demonstrate the usefulness of decomposition for integrated assessment models posed in a complementarity format. First, the complementarity formulation *cum* decomposition permits a precise representation of post-terminal damages thereby substantially reducing the model horizon required to produce an accurate approximation of the infinite-horizon equilibrium. Second, and central to the economic assessment of climate policies, the complementarity approach provides a means of incorporating second-best effects that are not easily represented in an optimization model.

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## 1 Introduction

Integrated assessment modeling emerged in the mid-eighties as a new paradigm for interfacing science and policy concerning complex environmental issues. An integrated assessment model provides a framework combining complementary knowledge from various disciplines in order to derive insights into key questions of policy formulation. Integrated assessment models (IAMs) link mathematical representations of the natural system and the socio-economic system to capture cause-effect chains including feedback.[1]

Weyant et al. (1996) distinguish two broad classes of IAMs: *policy optimization models* which seek optimal policies, and *policy simulation models* which assess specific policy measures. Policy optimization models are normative in the sense that they strive to derive an "ideal" policy, usually defined from an economic efficiency point of view.[2] The level of modeling detail in optimization models is constrained by the need to keep the optimization algorithm tractable. Therefore, these models tend to be based on compact representations of both the socio-economic and natural science systems. A prominent example of an optimizing IAM is the Dynamic Integrated Climate Economy (DICE) model by Nordhaus (1994) which incorporates stylized representations of both the global economy and the global carbon cycle. Policy evaluation models – often referred to as simulation models – typically are used to evaluate the impact of an exogenously specified policy. Avoiding optimization, policy evaluation models tend to

[1] An early example of formal integrated assessment is the RAINS model of acidification in Europe (Alcamo et al. 1985). Over the past years, a variety of models have been developed for the integrated assessment of climate change – for surveys see Weyant et al. (1996), Parson and Fisher-Vanden (1997), or Kelly and Kolstad (1999).

[2] Policy instrument variables such as emission control rates or emission taxes are derived given explicit policy goals, e.g. maximizing social welfare or minimizing the social costs of meeting exogenous environmental targets.

be descriptive and can contain much greater modeling detail on bio-/geo-physical, economic or social aspects. An early example of this type of model is the Integrated Model to Assess the Greenhouse Effect (IMAGE) by Rotmans (1990). The present paper focuses exclusively on policy optimization formulations.

In terms of policy design, optimization models are typically phrased as nonlinear mathematical programs (NLP) which permit derivation of best-response policies. Policy responses in these models can be traced to the rational behavior of economic agents. One shortcoming of the optimization approach is that computational tractability demands highly simplified formulations of both the economic and environmental sub-models. A more subtle disadvantage of IAMs cast as nonlinear programs is that they cannot directly incorporate second-best effects such as pre-existing tax distortions. Thus, "optimal" policies emerging from IAMs in NLP format are only optimal in a perfect, undistorted economy.

We present a decomposition approach to integrated assessment modeling of climate change that enables us to conveniently formulate the economic sub-model as a mixed complementarity problem (MCP – see Rutherford 1995). The MCP formulation overcomes two central shortcomings of the conventional nonlinear optimization approach. First, we can use superior terminal methods for approximating the infinite horizon in the economic model, which drastically reduces the number of model periods vis-à-vis a NLP approach, thereby increasing the scope for policy-relevant details on other model dimensions. Second, the MCP framework provides a means of incorporating second-best effects so that relevant complexities such as distortionary taxes or other market failures (e.g. knowledge spillovers) can be accounted for in the policy design process. As an added benefit – independent of the concrete mathematical MCP or NLP representation – our decomposition permits a convenient division of work between expert modelers in different disciplines.

The remainder of this paper is as follows. In section 2, we lay out the generic decomposition approach and provide the MCP formulation of terminal constraints for approximating the infinite horizon of the decomposed IA problem. In section 3, we first demonstrate the usefulness of the decomposed MCP framework for approximating the infinite horizon of the DICE model which has served for several years as a prototype IAM in the field of climate change. We then extend the basic DICE setting for public goods funded through distortionary taxation in order to illustrate the importance of a second-best setting for the derivation and design of climate policies. In section 4, we conclude. For the sake of brevity, we abstain from presenting a detailed description of the models' algebra. The interested reader can download this information together with the programming code for the numerical models from /texttthttp://debreu.colorado.edu/dice.pdf.

## 2 Decomposition

Figure 1 illustrates the generic structure of IAMs for climate policy analysis. These models aim to represent the causal chain through which (i) economic activities trigger anthropogenic greenhouse gas emissions, (ii) emissions of greenhouse gases translate into atmospheric concentration, temperature shift, and climate change, and (iii) climate change feeds back via the ecosystem to the economy.

Policy optimization models of climate change adopt a cost-benefit perspective in which the current marginal costs of controlling greenhouse gas emissions are balanced against the future marginal damages induced by those emissions. Climate change impacts are portrayed by para-

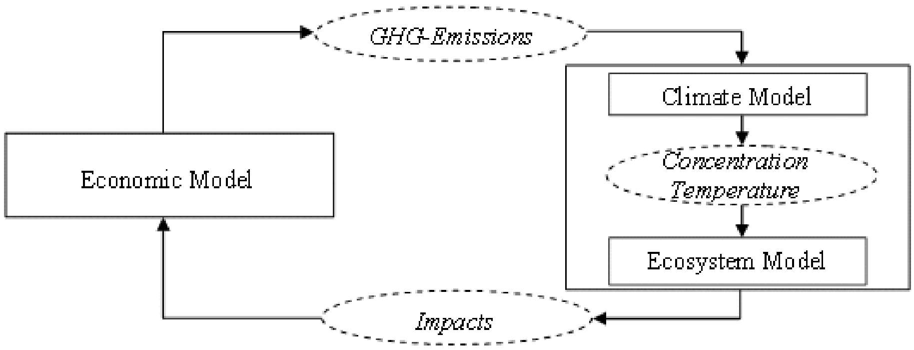


Figure 1: *Schematic Structure of Integrated Assessment Models for Climate Change*

metric relationships between economic losses and the global mean temperature (i.e., a "damage function").

In simple formal terms, the climate policy problem can be stated as a nonlinear optimization problem (NLP) of a single infinitely-lived agent:

$$\sum_{t=0}^{\infty} \rho_t U(C_t, D_t)$$

s.t.

$$\begin{aligned} C_t &= F(K_t, D_t, E_t) - I_t \\ K_{t+1} &= (1-\delta)K_t + I_t \\ D_t &= H(S_t) \\ S_{t+1} &= G(S_t, E_t) \\ K_0 = \bar{K}_0, & \qquad S_0 = \bar{S}_0 \end{aligned}$$

where $\rho_t$ is the discount factor in period $t$, $U$ denotes intertemporal utility, $C_t$ represents consumption in period $t$, $F$ characterizes production in period $t$ as a function of capital, damages (with potentially adverse effects on productivity), and emissions, $D_t$ denotes damages of climate change in period $t$, $K_t$ is the capital stock in period $t$ (with $K_0 = \bar{K}_0$ exogenously specified), $E_t$ denotes the emissions in period $t$, $I_t$ is investment in period $t$, $H$ describes the functional relationship between the climate state and damages, $S_t$ is a vector of the climate state (with $S_0 = \bar{S}_0$ as the initial climate state), and $G$ characterizes the motion of the climate state as a function of the previous climate state and anthropogenic emissions used as production input. Note that we can merge the relationships $D_t = H(S_t)$ and $S_{t+1} = G(S_t, E_t)$ into a single equivalent equation

$$D_t = \Gamma_t(S_0, E_0, E_1, ..., E_{t-1}),$$

where $\Gamma_t$ renders damages in period $t$ as a function of the initial climate state and emissions in all preceding periods.

Our decomposition is based on a linear approximation of the climate response, i.e. climate impacts $D_t$, to anthropogenic activities, i.e. emissions, of the economic system:

$$D_t \approx \bar{D}_t + \sum_{\tau=0}^{t} \frac{\partial \Gamma_t}{\partial E_\tau}(E_\tau - \bar{E}_\tau)$$

where $\bar{D}_t$ is the reference level value for climate impacts in period $t$, $\bar{E}_\tau$ is the reference level value for emissions in period $\tau$, $\frac{\partial \Gamma_t}{\partial E_\tau}$ denotes the gradient of climate impacts in period $t$ to anthropogenic emissions in period $\tau$.

In our implementation, we have evaluated the Jacobian $\frac{\partial \Gamma_t}{\partial E_\tau}$ for the climate sub-model using numerical differencing:[3]

$$\frac{\partial \Gamma_t}{\partial E_\tau} = \frac{\bar{D}_t - \Gamma_t(S_0, E_0, ..., \bar{E}_\tau + \epsilon, ..., \bar{E}_t)}{\epsilon}.$$

The climate model is nonlinear, so iterative refinement of the linear approximation is required. For our concrete numerical implementation of the DICE model, we find that this diagonalization procedure quickly converges.

A central advantage of the decomposition relates to the different nature of dynamics in the economic and the climate sub-models. Due to intertemporal optimization by economic agents, the economic sub-model must typically be solved simultaneously: current investment depends on future returns to capital, future economic damages, etc. In contrast, the climate sub model may be evaluated *recursively* given emission paths from the economic model. This permits us to solve the climate equations "off-line". The decomposition is effective provided that the climate system Jacobian is stable. Our computational experience suggests that this is the case, and this permits us to avoid integrating the complex system of climate system equations within the intertemporal economic model. Our decomposition then results in a sparse economic policy model based on simple but accurate reduced-form representation of climate impacts: We replace the explicit representation $S_t$ of the climate sub-model by a linear approximation of climate impacts $D_t$.

The reduced-form representation of the climate sub-model in our decomposition approach allows us to conveniently formulate the economic policy problem as a mixed complementarity problem (MCP). The MCP framework exploits the complementarity features of economic equilibrium, thereby including the NLP representation of economic equilibrium as a special case (Mathiesen 1985, Rutherford 1995).[4] As compared to the conventional representation of the climate policy problem in terms of a nonlinear program, the MCP formulation of the economic sub-model offers considerable advantages. First, we are better able to approximate the infinite horizon by state-variable targeting for the economic sub-model and cost-benefit calculus through the climate sub-model. Second, the MCP formulation relaxes the integrability constraints imposed by the NLP framework, thereby accommodating second-best settings that reflect initial inefficiencies.

### Terminal Constraints

Approximation of an infinite horizon economy by means of a *finite horizon* numerical model involves application of "terminal constraints". For example, in the steady state, gross investment

[3]Numerical differencing may pose high computational costs if the underlying climate model is computationally intensive. In those cases, a more sophisticated method of sensitivity analysis would be required.

[4]By forming the Lagrangian and differentiating, a nonlinear program can be posed as a complementarity problem.

is proportional to the capital stock through the growth rate of the labor force and the capital depreciation rate. A terminal constraint for investment might then require terminal investment to cover growth plus depreciation:

$$I_T = (\gamma + \delta) K_T$$

where $\gamma$ denotes the steady-state growth rate.

While this primal constraint seems perfectly reasonable, it introduces unintended adverse effects through the associated reduced cost associated with the terminal capital stock. This effect can be offset by a term in the utility function accounting for the "consumption" value of the terminal capital stock. After a policy shock, however, the equilibrium value of the capital stock in the terminal period is unknown.[5] A complementarity formulation, on the other hand, accomodates the representation of post-terminal capital stock as an endogenous variable. Using *state variable targeting* for this variable, the growth of investment in the terminal period is related to the growth rate of capital or any other "stable" quantity variable in the model (Lau, Pahlke, and Rutherford 2002), e.g.:

$$\frac{I_T}{I_{T-1}} = 1 + \gamma.$$

Beyond state variable targeting to determine the post-terminal capital stock, the decomposition *cum* MCP accommodates the precise approximation of post-terminal damages from emissions reflected by the terminal value of the climate state $S_T$. The complementarity model formulation has explicit price indices representing the cost of abatement and the benefits offered through abatement. A linear approximation to the climate model portrays the time profile of marginal benefits associated with emission reductions during and beyond the economic model horizon. Thus, we can compare the benefits associated with cutbacks in emissions in the later periods of the model with the benefits of those cutbacks in periods which lie beyond the terminal period of the model:[6]

$$-p_t \frac{\partial F}{\partial E_t} = \sum_{\tau=t}^{\infty} \frac{\partial \Gamma_\tau}{\partial E_t} p_\tau^D = \sum_{\tau=t}^{T} \frac{\partial \Gamma_\tau}{\partial E_t} p_\tau^D + \sum_{\tau=T+1}^{\infty} \frac{\partial \tilde{\Gamma}_\tau}{\partial E_t} \tilde{p}_\tau^D$$

where $p_t$ is the price of macro good production in period $t$, and $p_\tau^D$ is the price (cost) of damage in period $\tau$.

Post-terminal damages are calculated on the basis of the climate sub-model which is solved for several decades beyond the terminal period of the economic sub-model. Extrapolating present value prices and quantities into the post-terminal period then permits us to relate marginal costs of emission abatement throughout the time horizon to marginal damages occurring after the terminal period of the economic sub-model. The valuation of post-terminal damages is based on a geometric extrapolation of post-terminal prices $\tilde{p}_\tau^D$, and post-terminal climate $\tilde{\Gamma}_\tau$ is calculated on the basis of post-terminal emission paths which are extrapolated from the economic sub-model.

[5] The projected value of capital earnings in the post-terminal period can be estimated on the basis of dual multipliers in the NLP solution, but these multipliers and unavailable during the optimization process.

[6] In contrast, the optimization formulation of IAMs for climate change employs "transversality" adjustment terms to reflect post-terminal damages, but the specification of the values for these penalties remains ad-hoc (Nordhaus 1994).

### Integrability Constraints

First-order conditions of mathematical programs only correspond to equilibrium conditions for the case of integrability that implies efficient allocation (Pressman 1970 or Takayama and Judge 1971)[7]. Thus, IAMs of climate change cast as nonlinear optimization models are forced to provide a highly stylized representation of the economy in order to avoid "non-integrabilities" that can not be handled in the single optimization framework.[8] In contrary, the MCP formulation of economic problems permits the incorporation of "non-integrabilities" to reflect inefficiencies of market allocation induced by distortionary taxes, institutional price constraints, spillovers, etc.[9]

## 3 Illustration

We illustrate the advantages of our decomposed MCP formulation using the DICE model (Nordhaus, 1994) that is originally formulated as a nonlinear program. Because of its simplicity and relative transparency, DICE and its multiregional extension, RICE (Nordhaus and Yang 1996), have been widely used for the integrated assessment of climate change. DICE is based on Ramsey's model of saving and investment. A single world producer-consumer chooses between current consumption, investment in productive capital, and costly measures to reduce current emissions and slow climate change. Population growth and technological change (productivity growth) are both exogenous. The representative consumer maximizes the discounted utility of consumption over an infinite horizon subject to a Cobb-Douglas production function which includes damages from climate change as a quadratic function of global mean temperature. In the absence of abatement measures, anthropogenic emissions occur in direct proportion to output. Emissions per unit output are assumed to decline exogenously at a fixed rate and can be further reduced by costly emission-control measures. Within a simple reduced form "two-box" (ocean and atmosphere) climate sub-model based on Schneider and Thompson (1981), emissions accumulate and increase the stock of greenhouse gases in the atmosphere. As this stock grows, it increases the amount of solar radiation trapped by the earth's atmosphere which in turn triggers an increase in global mean temperature.

For our illustrative application of the decomposition approach, we distinguish two alternative mathematical formulations of the DICE integrated assessment model: the familiar implementation as a nonlinear mathematical program (NLP) and the model's representation as a mixed complementarity problem (MCP).

In order to evaluate the sensitivity of the optimal policy with respect to the model horizon, we run both models for horizons of 5, 10, 20, and 40 periods (with each period representing a 10-year time interval). As is evident in Figure 2, the MCP model is virtually insensitive to the model horizon, whereas the NLP model shows a drastic sensitivity, in particular for the first few decades. Furthermore, the differences in optimal emission control rates[10] between the two model formulations differ substantially, particularly for short time horizons. In practical terms,

[7] In practical terms, integrability refers to a situation where the shadow prices of programming constraints coincide with market prices.

[8] Integrability problems may be relaxed in the optimization context by adding a correction term to the objective and solving a sequence of nonlinear programs to obtain a market equilibrium (see e.g. Rutherford 1999).

[9] Other important examples of non-integrabilities include individual demand functions which do not only depend on prices but also on the initial endowments (Chipman 1974).

[10] The key policy instrument in the DICE model is the emissions control rate, the fraction of emissions which are mitigated relative to the uncontrolled level.

the precise terminal approximation of the MCP approach offers a major improvement in the range and details of policy analysis that can be covered: Since the economic sub-model only requires a short-term horizon, one can elaborate on policy-relevant complexities.

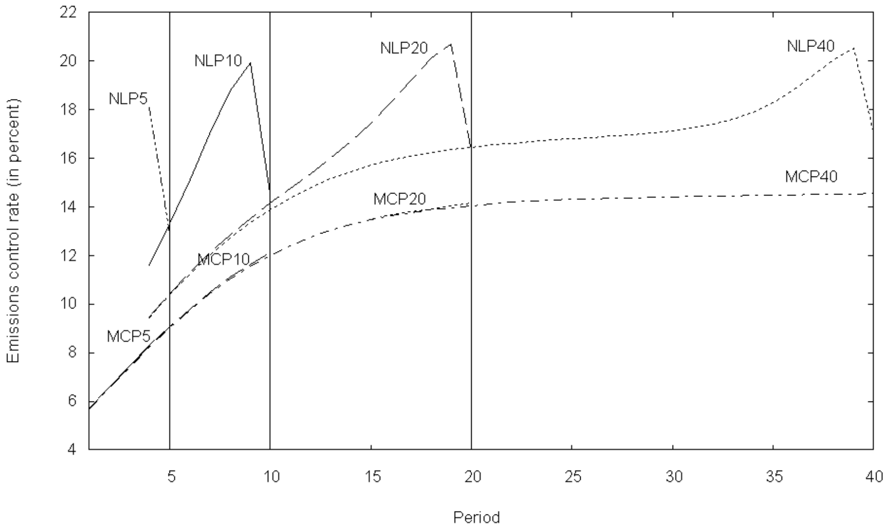


Figure 2: *Sensitivity of Emission Control Rate with respect to the Model Horizon - NLP Model vs. MCP Model*

Another key advantage of the decomposed MCP framework for applied policy analysis is the ease with which it can incorporate second-best effects. We illustrate the importance of market distortions by considering a simple extension of the DICE model in which a public good provided in each period is funded through a distortionary tax on capital earnings. In the reference simulation, we hold the capital tax fixed at an exogenous rate and compute the "optimal" abatement profile together with the resulting level of public goods provision. In the counterfactual simulation, we endogenize the capital tax rate through an equal-yield constraint (keeping public good provision at the reference level) and evaluate the marginal utility of perturbations of the "optimal" abatement profile for each model period.

As has been observed by several authors (Goulder 1995) preexisting tax distortions affect the economic cost of climate policy instruments. When the government applies emission restrictions, these raise revenue which may be used to reduce other taxes. In this case, where revenues from carbon permit sales are used to replace distortionary taxes, the "optimal" abatement profile is too low. This occurs because the marginal benefit calculus is implicitly based on a marginal cost of public funds equal to 1, whereas distortionary financing of public provision implies that the marginal cost of public funds is greater than one. The larger the baseline tax rate on capital in our example, the larger is the marginal benefit of increasing stringency of environmental restrictions. Figure 3 illustrates our reasoning for alternative capital tax rates of 5%, 10%, 25% and 50%.

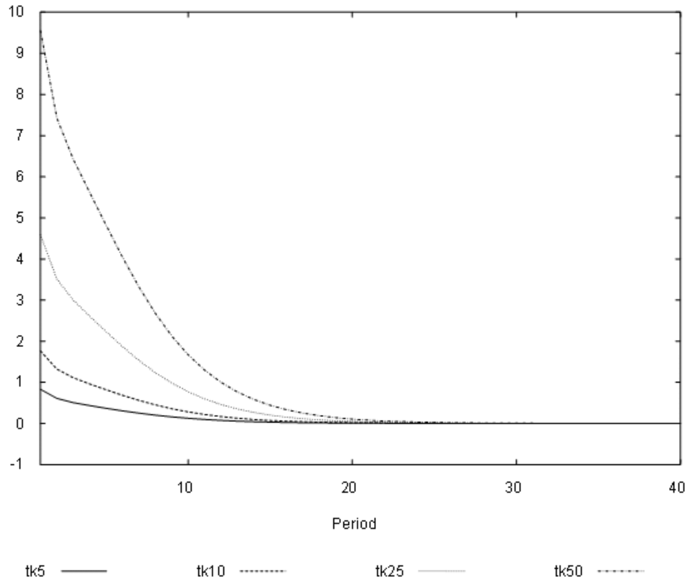


Figure 3: *Marginal Utility of 1% Additional Abatement For Alternative Capital Tax Rates*

# 4 Conclusions

In this paper, we have presented a new approach to integrated assessment modeling of climate change. Our decomposition of IAMs is based on a linear approximation to the climate sub-model and provides a convenient framework for the complementarity formulation of the economic sub-model. This offers considerable advantages as compared to traditional nonlinear programming. First, the complementarity formulation *cum* decomposition permits more precise terminal approximation using state-variable targeting for the economic sub-model. It also permits more accurate cost-benefit calculus based on a climate sub-model operating over an extended time horizon. From a computational point of view, the reduction in model periods vis-à-vis nonlinear programming permits more scope for policy-relevant details. Second, the MCP formulation provides a convenient means of incorporating second-best effects that may substantially alter policy conclusions based on the assumptions of perfectly undistorted economies.

Beyond the specific advantages of the complementarity approach over nonlinear programming, our decomposition allows the separation of components from different disciplines through a consistent, well-defined interface. The economic model generates emission paths, and the climate model returns temperature profiles and their partial derivatives with respect to emissions. In this way, modelers in each discipline can focus on their specific expertise. Furthermore, the decomposition permits assessment of the relative importance of the various model components – it becomes e.g. fairly easy to ex-change the natural science modules and track down the sensitivity of results with respect to alternative formulations of natural science relationships.

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