

# Tools for Building National Economic Models Using State-Level IMPLAN Social Accounts

Sebastian Rausch

MIT Joint Program on the Science and Policy of Global Change

Massachusetts Institute of Technology

Thomas F. Rutherford

Swiss Federal Institute of Technology (ETH), Zürich

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## **Abstract**

This paper describes a set of GAMS programs for building national economic models for the United States based on data files from the Minnesota IMPLAN Group. The IMPLAN data includes a set of benchmark economic data encompassing input-output tables with 508 commodities, nine classes of private households and six types of government agents. The GAMS programs provided here permit flexibility in the aggregation and organization of datasets for economic analysis. This package is designed to operate with a complete national package, consisting of social accounting matrices for each of the 50 states. The distribution archive includes an IMPLAN state-level model to illustrate the accounting identities in the underlying data and a prototype regional small open economy model for the US that can be used as a starting point for further modeling tasks.

# 1 Introduction

The IMPLAN (IMpact analysis for PLANning) data includes a set of benchmark economic data for the United States which covers each of the 50 states encompassing input-output tables with 508 commodities, nine classes of private households and six types of government agents. The system of IMPLAN Social Accounting Matrices (SAM) can provide applied general equilibrium (AGE) models with consistent calibration data and therefore facilitates the development of effective quantitative tools to evaluate the macroeconomic and distributional effects of federal and state policies by region, industry, household and government institution.

The purpose of this paper is to provide (documentation of) a set of GAMS (General Algebraic Modeling System) programs for building regional economic models for the United States based on state-level IMPLAN social accounts from the Minnesota IMPLAN Group.<sup>1</sup> The GAMS programs provided here include ancillary tools for the translation of the IMPLAN data files into GAMS readable form, and GAMS programs for dataset aggregation and reconciliation for economic analysis. The distribution archive also includes two illustrative GAMS models formulated as mixed complementarity problems in GAMS/MCP and GAMS/MPSGE format: (i) a static small open economy state-level IMPLAN model to indicate the nature of accounting identities in the underlying data and (ii) a prototype small open economy regional US model. However, due to their highly stylized and illustrative character the models are not intended to be used "as-is" for policy analysis. The tools and models provided here are designed to permit flexibility in the aggregation and organization of datasets and thus provide a general starting point for the development of more specific research questions. This package is designed to operate with a complete national package of the most recent 2006 IMPLAN data, consisting of social accounting matrices for each of the 50 states.

The remainder of this paper is organized as follows. Section 2 briefly describes the IMPLAN dataset. Section 3 organizes the benchmark IMPLAN data in a small open economy framework and illustrates the accounting identities by means of a simple state-level small open economy model. Section 4 formulates a prototype US regional model based on IMPLAN data and describes its implementation in GAMS/MCP and GAMS/MPSGE. Section 5 has a practical perspective with step-by-step instructions on how to install the IMPLANinGAMS package and with an overview of ancillary GAMS programs used for reading,

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<sup>1</sup> For more information on the Minnesota IMPLAN Group see [www.implan.com](http://www.implan.com).

filtering and aggregation of the data. We hope that this provides a short learning curve for economists who wish to perform a few calculations using the IMPLAN dataset.

## 2 The IMPLAN Dataset

Minnesota IMPLAN Group, Inc. or MIG, Inc was founded in 1993 by Scott Lindall and Doug Olson as an outgrowth of their work at the University of Minnesota starting in 1984. Originally concerned with providing input-output data on forest services, the IMPLAN data was extended in 1995 to provide a system of comprehensive Social Accounting Matrices comprising transactions that occur between producers, and intermediate and final consumers, and non-market transactions, such as transfer payments between institutions. The most recent 2006 version of the IMPLAN data includes a set of benchmark economic data for each of the 50 states encompassing input-output tables with 508 commodities, 9 classes of private households and 6 types of government agents. IMPLAN data is not freely available and may be obtained from Minnesota IMPLAN Group.

Comprehensive and detailed documentation of the IMPLAN dataset –including definitions of accounts and the various types of data sources used for the construction of the data– is available on the webpage of Minnesota IMPLAN group at [www.implan.com](http://www.implan.com).<sup>2</sup> Nonetheless, and to prepare the ground for the subsequent discussion of GAMS tools and models, the basic structure of a regional IMPLAN SAM shall be briefly described here.

A SAM represents a data system in the form of a square matrix which brings together data on production and income generation as generated by different institutional groups and classes, on the one hand, and data about expenditure of these incomes by them on the other. Table 1 shows the basic structure of a regional SAM in the IMPLAN framework. The SAM tracks the dollar flows through the economy as sets of income and payments. Each row cell represents an institutional or industry receipt of income. Each column cell represents an institutional or industry payment or disbursement. *Micro-consistency* is reflected by the fact that row and column sums balance.

The SAM structure shown in Table 1 refers to the highest level of aggregation. Each cell of the SAM matrix represents a sub-matrix. Table 2 displays the set indices and respective elements for the row and column header of each sub-matrix.

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<sup>2</sup> A good reference to start with is the User-Analysis-Data Guide, [Minnesota Implan Group \[2004\]](#).

TABLE 1: REGIONAL SOCIAL ACCOUNTING MATRIX IN THE IMPLAN FRAMEWORK

|              | Industry           | Commodity    | Factors             | Institutions       | Trade          |
|--------------|--------------------|--------------|---------------------|--------------------|----------------|
| Industry     |                    | <i>Make</i>  |                     |                    | <i>Exports</i> |
| Commodity    | <i>Use</i>         |              |                     | <i>Consumption</i> |                |
| Factors      | <i>Value Added</i> |              |                     |                    | <i>Exports</i> |
| Institutions |                    | <i>Sales</i> | <i>Transfers</i>    | <i>Transfers</i>   | <i>Exports</i> |
| Trade        | <i>Imports</i>     |              | <i>Factor Trade</i> | <i>Imports</i>     | <i>Exports</i> |

### 3 Benchmark Data and Accounting Identities

This section describes a simple state-level small open economy (SOE) accounting model which serves to illustrate the accounting identities in the IMPLAN dataset. GAMS code model for the accounting model is provided in Appendix A and the identities presented below can be verified running the GAMS/MPSGE program `implan_acc.gms` which is located in the model directory of this package.

Figure 1 provides a flow chart which illustrates the implicit accounting identities.  $Y_s$  and  $YS_s$  portray the production and total supply of commodity  $s$ , respectively.  $M_s$  portrays imports of good  $s$  into region  $r$ .  $CD_h$ ,  $ID_h$ , and  $GD_h$  portray private consumption, investment and public demand, respectively.  $RH_h$  and  $RG_{pub}$  stand for representative household and government consumers.  $Firms_r$  portrays representative firms.<sup>3</sup>

In this figure domestic and imported goods markets are represented by horizontal lines at the top of the figure. Domestic production ( $vom_s$ ) and goods supply by institutions ( $evpm_{s,i}$ ) are distributed to exports ( $vxm_{s,trd}$ ), intermediate demand ( $vd fm_{g,s}$ ), household consumption ( $vd pm_{s,h}$ ), investment ( $vd im_s$ ) and government consumption ( $vd gm_{s,pub}$ ). Exports comprise sectoral and institutional exports. The accounting identity in the IMPLAN dataset for domestic output is therefore:

$$\begin{aligned}
 vom_s + \sum_i evpm_{s,i} &= \sum_{trd} vxm_{s,trd} + \sum_g vdfm_{g,s} \\
 &+ \sum_h vdpm_{s,h} + vdim_s + \sum_{pub} vdgm_{s,pub}.
 \end{aligned}$$

Imported goods which have an aggregate value of  $vim_{s,trd}$  enter intermediate demand

<sup>3</sup> The following exposition pertains to the state level and hence a state or region index is omitted.

TABLE 2: SET INDICES AND ELEMENTS

| SET        | SUBSET         | ELEMENT            | DESCRIPTION  |
|------------|----------------|--------------------|--|
| <i>s,g</i> |                |                    | <b>Industries/commodities:</b>                           |
|            |                | (...) <sup>1</sup> | An aggregation of the 508 sectors in the IMPLAN database |
| <i>f</i>   |                |                    | <b>Factors:</b>  |
|            |                | <i>empl</i>        | Employee Compensation (5001)                             |
|            |                | <i>prop</i>        | Proprietary Income (6001)                                |
|            |                | <i>othp</i>        | Other Property Income (7001)                             |
|            |                | <i>btax</i>        | Indirect Business Taxes (8001)                           |
| <i>i</i>   |                |                    | <b>Institutions:</b>                                     |
|            | <i>h(i)</i>    |                    | Private (household) institutions                         |
|            |                | <i>hhl</i>         | Households less than 5k (10001)                          |
|            |                | <i>hh5</i>         | Households 5-10k (10002)                                 |
|            |                | <i>hh10</i>        | Households 10-15k (10003)                                |
|            |                | <i>hh15</i>        | Households 15-20k (10004)                                |
|            |                | <i>hh20</i>        | Households 20-30k (10005)                                |
|            |                | <i>hh30</i>        | Households 30-40k (10006)                                |
|            |                | <i>hh40</i>        | Households 40-50k (10007)                                |
|            |                | <i>hh50</i>        | Households 50-70k (10008)                                |
|            |                | <i>hh70</i>        | Households 70k+ (10009)                                  |
|            | <i>pub(i)</i>  |                    | Public (government) institutions                         |
|            |                | <i>fnd</i>         | Federal Government NonDefense (11001)                    |
|            |                | <i>fdf</i>         | Federal Government Defense (11002)                       |
|            |                | <i>fin</i>         | Federal Government Investment(11003)                     |
|            |                | <i>sln</i>         | State Local Govt NonEducation (12001)                    |
|            |                | <i>sle</i>         | State Local Govt Education (12002)                       |
|            |                | <i>sin</i>         | State Local Govt Investment(12003)                       |
|            | <i>corp(i)</i> |                    | Corporate institutions                                   |
|            |                | <i>ent</i>         | Enterprises (Corporations) (13001)                       |
|            |                | <i>inv</i>         | Gross Private Fixed Investment (Capital) (14001)         |
|            |                | <i>stk</i>         | Inventory Additions Deletions (14002)                    |
| <i>trd</i> |                |                    | <b>Trade:</b>  |
|            |                | <i>ftrd</i>        | Foreign Trade  |
|            |                | <i>dtrd</i>        | Domestic Trade   |
| <i>r</i>   |                |                    | <b>Regions:</b>  |
|            |                | (...)              | An aggregation of the 50 states in the IMPLAN database   |

*Note:* 1: A list of all 508 IMPLAN industries can be found in Appendix A, [Minnesota Implan Group \[2004, pp. 291-301\]](#). The figures in parentheses refer to the IMPLAN type code number.

( $vifm_{s,itrd,g}$ ), private consumption ( $vipm_{s,trd,h}$ ), public consumption ( $vigm_{s,trd,pub}$ ) and investment demand ( $viim_{s,trd}$ ). The accounting identity for these flows is thus:

$$vim_{s,trd} = \sum_g vifm_{s,trd,g} + \sum_h vipm_{s,trd,h} + \sum_{pub} vigm_{s,trd,pub} + viim_{s,trd}.$$

Inputs to  $Y_s$  include intermediate inputs (domestic and imported) and factors of production ( $vfm_{f,s}$ ). Factor earnings accrue to all institutions. Factor market equilibrium is given by an identity relating the value of factor payments to factor income ( $evom_{f,i}$ ):

$$\sum_s vfm_{f,s} = \sum_i evom_{f,i}.$$

The market clearance condition for foreign exchange relates transfers by institution ( $vtrn_i$ ) and aggregate exports to aggregate imports:

$$\sum_i vtrn_i + \sum_s \sum_{trd} vxm_{s,trd} = \sum_s \sum_{trd} vim_{s,trd}.$$

Budget constraints for the nine groups of representative households and six types of government agents relate factor income to consumption expenditure ( $evom_{f,i}$ ), profits ( $vprf_i$ ), institutional goods supply ( $evpm_{s,i}$ ) and inter-institutional transfers ( $vtrn_i$ ) to consumption expenditure ( $vpm_i$ ):

$$\sum_f evom_{f,i} + vprf_i + \sum_s evpm_{s,i} + vtrn_i = vpm_i \quad , i \in \{h, pub\}.$$

The budget constraint for representative firms relates factor income, goods supply and transfers to “expenditure” spent for profits consumption and investment demand ( $vinv$ ):

$$\sum_f evom_{f,i} + \sum_s evpm_{s,i} + vtrn_i = vprf_i + vinv \quad , i \in \{corp\}.$$

To this point we have outlined two types of consistency conditions which are part of the IMPLAN database: market clearance (supply = demand for all goods and factors, and income balance (net income = net expenditure) conditions. A third set of identities involve net operating profits by all sectors in the economy. In this accounting framework production is assumed to take place under perfect competition with constant returns to scale, hence there are no excess profits, and the cost of inputs must equal the value of outputs. This

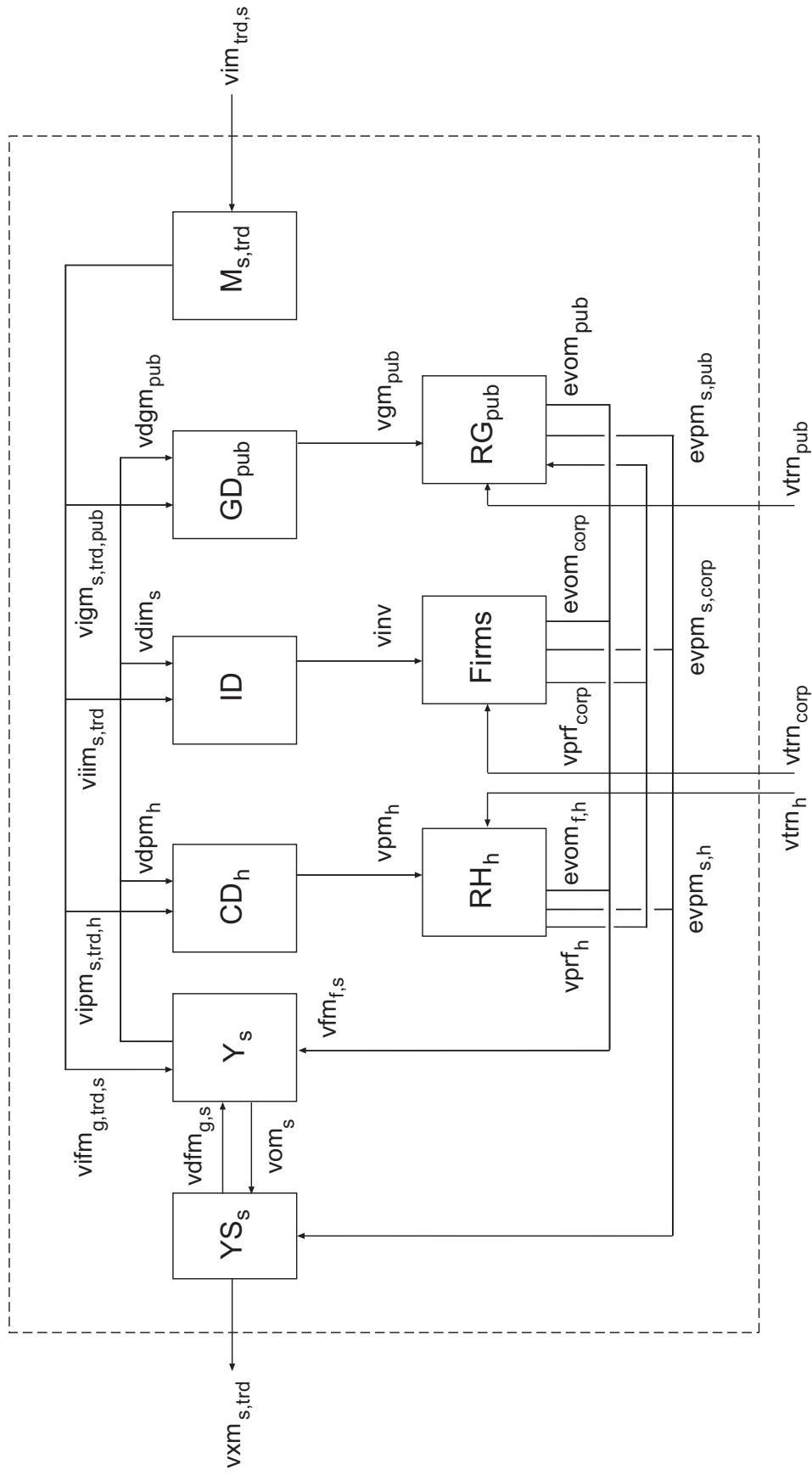


FIGURE 1: REGIONAL ECONOMIC STRUCTURE

condition applies for each of the production sectors:

$$\begin{aligned}
Y_s : \quad & \sum_g vdfm_{g,s} + \sum_{trd} \sum_g vifm_{g,trd,s} + \sum_f vfm_{f,s} = vom_s, \\
CD_h : \quad & \sum_s vdpm_{s,h} + \sum_{trd} \sum_s vipm_{g,trd,h} = vpm_h, \\
ID : \quad & \sum_s vdim_s + \sum_{trd} \sum_s viim_{s,trd} = vinv, \\
GD_{pub} : \quad & \sum_s vdgm_{s,pub} + \sum_{trd} \sum_s vigm_{s,trd,pub} = vgm_{pub}.
\end{aligned}$$

## 4 A Prototype US Regional Model

This section lays out a prototype static regional CGE model for the US economy which is based on the complete national IMPLAN dataset. The model is highly stylized and not intended to be used "as-is" for policy analysis. Despite its simplicity it provides a starting point for further model development based on the IMPLAN data.

The US economy is modeled adopting the small open economy assumption, i.e., export and import prices in foreign currency are not affected by the behavior of the domestic economy. In other words, the small open economy faces infinitely elastic world export demand and world import supply functions. Foreign trade is modeled according to the standard assumption that foreign and domestic goods are imperfect substitutes.<sup>4</sup> When modeling intra-national (or domestic) trade, commodities produced in different regions are assumed to be homogeneous.

GAMS code for the prototype model is provided in GAMS/MCP and GAMS/MPSGE format in Appendix B and C, respectively, and corresponding GAMS files `soe_mge.gms` and `soe_mcp.gms` can be found in the model directory of this package. To run these programs, IMPLANinGAMS has to be installed first and an aggregated data set in which all domestic trade flows balance has to be constructed. For instructions, refer to Section 5.

### 4.1 The Primal Formulation

The notation used in the model is summarized in the Tables 2-4. Table 2 defines the various dimensions which characterize an instance of the model, including the set of sectors and

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<sup>4</sup> Without this last assumption only the net trade balance would matter and the levels of imports and exports would be indeterminate.



TABLE 3: ACTIVITY LEVELS

| VARIABLE       | DESCRIPTION                         |
|----------------|-------------------------------------|
| $Y_{s,r}$      | Sectoral production                 |
| $A_{s,r}$      | Armington aggregation               |
| $C_{h,r}$      | Consumption by household            |
| $GOV_{pub,r}$  | Public output                       |
| $INV_r$        | Investment                          |
| $RH_{h,r}$     | Income of representative households |
| $GOVT_{pub,r}$ | Income of government agents         |
| $TAXREV$       | Income of tax revenue agent         |

TABLE 4: PRICES

| VARIABLE       | DESCRIPTION                        |
|----------------|------------------------------------|
| $py_{s,r}$     | Sectoral output prices             |
| $p_{s,r}$      | Price for domestic output          |
| $pa_{s,r}$     | Armington aggregate prices         |
| $pc_{h,r}$     | Consumption by household           |
| $pn_s$         | Intra-national trade price         |
| $pinv_r$       | New investment                     |
| $pgov_{pub,r}$ | Public output                      |
| $pf_{fa,r}$    | Factor prices                      |
| $px$           | Foreign exchange                   |
| $pcarb$        | Tradable CO2 emission permit price |
| $ptax_r$       | Business taxes                     |

commodities, the set of factors of production, the set of institutions and the set of regions. Table 3 defines the primal variables (activity levels) which define an equilibrium. Table 4 defines the relative price variables for goods and factors in the model. As is the case in any Arrow-Debreu model, the equilibrium conditions determine relative rather than nominal prices.

The IMPLAN database includes all 50 states and 508 commodities, but dimensionality typically limits the number of regions and goods which can be included in a single model. Datasets are easily aggregated which permits empirical applications to be formulated and debugging with small dimensional models. Section 5.5 explains how to employ different mappings for aggregation of regions, sectors, factors of production and economic agents.

The benchmark identities presented in the previous section indicate the market clearance, zero profit and income balance conditions but do not, however, characterize the be-

havior of agents in the model. In the competitive equilibrium setting, the standard assumption of optimizing atomistic agents applies for both producers and consumers. Profit maximization in the constant returns to scale setting is equivalent to cost minimization subject to technical constraints. For sector  $Y_{s,r}$  we characterize input choices as though they arose from minimization of unit costs: <sup>5</sup>

$$\min_{ddifm_{g,s,r}, dfm_{f,s,r}, demit} c_{s,r}^A + c_{s,r}^F + c^{CA} + c^{BT} \quad (1)$$

s.t.

$$c_{s,r}^A = \sum_g pa_{g,r} ddifm_{g,s,r}$$

$$c_{s,r}^F = \sum_{fa} pf_{fa,r} dfm_{fa,s,r}$$

$$c^{CA} = pcarb demit$$

$$c^{BT} = ptax dfm_{btax,s,r}$$

$$F_{s,r}(ddifm_{g,s,r}, dfm_{f,s,r}, demit) = Y_{s,r}.$$

The production function  $F(\cdot)$  is described by a nested constant-elasticity-of-substitution (CES) form which is shown in MPSGE syntax and indicated by `i :` fields in Figure 2.<sup>6</sup> The supply of output  $Y_{s,r}$  to domestic and trade (domestic and foreign) markets is portrayed as arising from the following profit-maximization problem:

$$\max_{sdmi_{s,r}, sxm_{s,trd,r}} p_{s,r} sdmi_{s,r} + pfx sxm_{s,trd,r} + pn_s sxm_{s,dtrd,r} \quad (2)$$

s.t.

$$Y_{s,r} = \Gamma_{s,r}(sdmi_{s,r}, sxm_{s,trd,r}).$$

The production function  $\Gamma(\cdot)$  is described by a constant-elasticity-of-transformation (CET) form which is shown in MPSGE syntax and indicated by `o :` fields in Figure 2.

The choice among imports from different trading partners (domestic, domestic and foreign trade) is based on Armington's idea of regionally differentiated products. This is re-

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<sup>5</sup> Decision variables appearing in the primal model correspond to the benchmark data structures with the initial "v" replaced by either "d" or "s". Hence, while  $vdfm_{g,s,r}$  represents benchmark intermediate demand for good  $g$  in the production of good  $s$  in region  $r$ ,  $ddfm_{g,s,r}$  ( $sdfm_{g,s,r}$ ) represents the corresponding intermediate demand (supply) variable in the equilibrium producer's decision problem.

<sup>6</sup> For an overview of the MPSGE framework and the syntax see Rutherford [1999].

FIGURE 2: PRODUCTION FUNCTION IN MPSGE SYNTAX

```

$prod:y(r,s)$vom(r,s) s:0 t:2 va:1
  o:p(r,s)          q:vdmi(r,s)
  o:pfx            q:vxm(r,s,"ftrd")
  o:pn(s)          q:vxm(r,s,"dtrd")
  i:pa(r,g)        q:vdifm(r,g,s)
  i:pf(r,fa)       q:vfmr(r,fa,s)   va:
  i:pcarb          q:emit(r,s)
  i:ptax(r)        q:vfmr(r,"btax",s)

```

flected by the following cost minimization problem:

$$\min_{ddmi_{s,r}, dim_{s,trd,r}} p_{s,r} ddmi_{s,r} + pfx dim_{s,ftrd,r} + pn_s dim_{s,dtrd,r} \quad (3)$$

s.t.  $F_{s,r}^A(ddmi_{s,r}, dim_{s,trd,r}) = A_{s,r}.$

The import aggregation function portrayed by  $F^A(\cdot)$  in (3) is described by the nested CES function shown in Figure 3.

FIGURE 3: ARMINGTON AGGREGATION IN MPSGE SYNTAX

```

$prod:a(r,s)$va(r,s) s:4 m:8
  o:pa(r,s)          q:va(r,s)
  i:p(r,s)           q:vdmi(r,s)
  i:pfx              q:vim(r,s,"ftrd")   m:
  i:pn(s)            q:vim(r,s,"dtrd")   m:

```

The investment demand is represented by a fixed coefficient (Leontief) aggregation of Armington goods. Its production can be portrayed by the following cost minimization problem:

$$\min_{dinvd_{s,r}} \sum_s pa_{s,r} dinvd_{s,r} \quad (4)$$

s.t.  $F_r^{INV}(dinvd_{s,r}) = INV_r,$

where the respective production function portrayed by  $F^{INV}(\cdot)$  is described by the Leontief function shown in Figure 4.

Similarly, public consumption in the model is a Leontief (linear) composite of Armington

FIGURE 4: INVESTMENT PRODUCTION IN MPSGE SYNTAX

```

$prod:inv(r)
  o:pinv(r)      q:vinv(r)
  i:pa(r,s)      q:vinvd(r,s)

```

goods. Its production can be portrayed by the following cost minimization problem:

$$\min_{ddgm_{s,pub,r}, digm_{s,trd}} \sum_s pa_{s,r} \left( ddgm_{s,pub,r} + \sum_{trd} digm_{s,trd,pub,r} \right) \quad (5)$$

s.t. 
$$F_{pub,r}^{GOV}(ddgm_{s,pub,r}, digm_{s,trd}) = GOV_{pub,r},$$

where the production function portrayed by  $F^{GOV}(\cdot)$  is described by the production block shown in Figure 5.

FIGURE 5: PUBLIC CONSUMPTION IN MPSGE SYNTAX

```

$prod:gov(r,pub)
  o:pgov(r,pub)  q:vgm(r,pub)
  i:pa(r,s)      q:(vdgm(r,s,pub)+sum(trd,vigm(r,s,trd,pub)))

```

Private consumption consistent with utility maximization is portrayed by minimization of the cost of a given level of aggregate consumption:

$$\min_{ddpm_{s,h,r}, dipm_{s,h}} \sum_s pa_{s,r} \left( ddpm_{s,h,r} + \sum_{trd} dipm_{s,trd,h,r} \right) \quad (6)$$

s.t. 
$$F_{h,r}^C(ddpm_{s,h,r}, dipm_{s,h}) = C_{h,r},$$

Final demand in the model is characterized by a Cobb-Douglas tradeoff across composite goods which include both domestic and imported inputs. The nested CD-CES function is displayed in Figure 6.

FIGURE 6: PRIVATE CONSUMPTION IN MPSGE SYNTAX

```

$prod:c(r,h) s:1
  o:pc(r,h)      q:vpm(r,h)
  i:pa(r,g)      q:(vdpm(r,g,h)+sum(trd,vipm(r,g,trd,h)))

```

## 4.2 Equilibrium conditions

We now define the general equilibrium of the model in a complementarity format.<sup>7</sup> Rutherford [1995] and Mathiesen [1985] have shown that a complementary-based approach is convenient, robust, and efficient. A characteristic of economic models is that they naturally involve a complementary problem, i.e. given a function  $F: \mathbb{R}^n \rightarrow \mathbb{R}^n$ , find  $z \in \mathbb{R}^n$  such that  $F(z) \geq 0$ ,  $z \geq 0$ , and  $z^T F(z) = 0$ .

An equilibrium in complementarity format is represented by a vector of activity levels, a non-negative vector of prices, and a non-negative vector of incomes such that:

1. no production activity makes a positive profit (zero profit conditions),
2. . excess supply (supply minus demand) is non-negative for all goods and factors (market clearance conditions), and
3. expenditure does not exceed income (budget constraints).

Zero-profit conditions exhibit complementary slackness with respect to associated activity levels, market clearing conditions exhibit complementary slackness with respect to market prices, and budget constraints define income variables.

To clarify exposition, we denote benchmark data parameters with a bar “—” to distinguish them from model variables.

### 4.2.1 Zero profit conditions

All production activities in the model are represented by constant-returns-to-scale technologies, and markets are assumed to operate competitively with free entry and exit. As a consequence, equilibrium profits are driven to zero and the price of output reflects the cost of inputs. The following sets of equations relating marginal cost (LHS) to output prices (RHS) are part of the definition of an equilibrium:

*Sectoral production* ( $Y_{s,r}$ ):

$$\theta_{s,r}^i ca_{s,r} + \theta_{s,r}^{fa} cf_{s,r} + \theta_{s,r}^e pcarb + \theta_{s,r}^t ptax_r = py_{s,r} \quad (7)$$

---

<sup>7</sup> In this simple version of the model we formulate all equilibrium conditions as equalities, i.e., all equilibrium prices and quantities are positive, and hence the complementary slackness aspect is trivial.

where

$$\begin{aligned}
cf_{s,r} &= \prod_{fa} (pf_{fa,r})^{\theta_{fr,r}} \\
ca_{s,r} &= \sum_g \theta_{g,r}^a pa_{g,s} \\
py_{s,s} &= \left( \theta_{s,r}^d p_{s,r}^{1+\eta} + \theta_{s,r}^{fx} pfx^{1+\eta} + \theta_{s,r}^{nt} pn_s^{1+\eta} \right)^{1/(1+\eta)}.
\end{aligned}$$

*Armington aggregation* ( $A_{s,r}$ ):

$$\left( \theta_{s,r} p_{s,r}^{1-\sigma_a} + \sum_{trd} \theta_{trd,s,r}^{ar} cfn_{s,r}^{1-\sigma_a} \right)^{1/(1-\sigma_a)} = pa_{s,s} \quad (8)$$

where

$$cfn_{s,r} = \left( \theta_{s,r}^{trd} pfx^{1-\sigma_t} + (1 - \theta_{s,r}^{trd}) pn_s^{1-\sigma_t} \right)^{1/(1-\sigma_t)}.$$

*Investment* ( $INV_r$ ):

$$\sum_s pa_{s,r} \overline{vinvd}_{s,r} = pinu_r \overline{vinu}_r. \quad (9)$$

*Public consumption* ( $GOV_{r,pub}$ ):

$$\sum_s pa_{s,r} \left( \overline{vdgm}_{s,pub,r} + \sum_{trd} \overline{vigm}_{s,trd,pub,r} \right) = pgov_{pub,r} \overline{vgm}_{pub,r}. \quad (10)$$

*Private consumption* ( $C_{r,h}$ ):

$$\prod_s (pa_{s,r})^{\theta_{s,h,r}^c} = pc_{h,r}. \quad (11)$$

The values shares from the benchmark data used in the zero profit conditions above are given by:

$$\begin{aligned}
\theta_{s,r}^i &= \frac{\sum_g \overline{vdifm}_{g,s,r}}{\overline{vdmi}_{s,r} + \sum_{trd} \overline{vxm}_{s,trd}}, \\
\theta_{s,r}^{fa} &= \frac{\sum_{fa} \overline{vfm}_{fa,s,r}}{\overline{vdmi}_{s,r} + \sum_{trd} \overline{vxm}_{s,trd}}, \\
\theta_{s,r}^e &= \frac{\overline{emit}_{s,r}}{\overline{vdmi}_{s,r} + \sum_{trd} \overline{vxm}_{s,trd}},
\end{aligned}$$

$$\theta_{s,r}^t = \frac{\overline{vfm}_{btax,s,r}}{\overline{vdm}_{s,r} + \sum_{trd} \overline{vxm}_{s,trd}},$$

$$\theta_{fr,r} = \frac{\overline{vfm}_{fa,s,r}}{\sum_{fd'} \overline{vfm}_{fd',s,r}},$$

$$\theta_{g,r}^a = \frac{\overline{vdifm}_{g,s,r}}{\sum_{g'} \overline{vdifm}_{g',s,r}},$$

$$\theta_{s,r}^d = \frac{\overline{vdm}_{s,r}}{\overline{vdm}_{s,r} + \sum_{trd} \overline{vxm}_{s,trd,r}},$$

$$\theta_{s,r}^{fx} = \frac{\overline{vxm}_{s,ftd,r}}{\overline{vdm}_{s,r} + \sum_{trd} \overline{vxm}_{s,trd,r}},$$

$$\theta_{s,r}^{nt} = \frac{\overline{vxm}_{s,dtrd,r}}{\overline{vdm}_{s,r} + \sum_{trd} \overline{vxm}_{s,trd,r}},$$

$$\theta_{trd,s,r}^{ar} = \frac{\overline{vdm}_{s,r}}{\overline{va}_{s,r}},$$

$$\theta_{s,r}^{ftd} = \frac{\overline{vim}_{s,ftd,r}}{\sum_{trd} \overline{vim}_{s,trd,r}},$$

$$\theta_{s,h,r}^c = \frac{\overline{vdp}_{s,h,r} + \sum_{trd} \overline{vip}_{s,trd,h}}{\sum_{s'} \overline{vdp}_{s',h,r}}.$$

#### 4.2.2 Market clearance conditions

Market clearance conditions exhibit complementary slackness with respect to prices. We make use of Shepard's Lemma to derive conditional demand from unit cost functions. Demand components are related to the notation used in the primal formulation of the model by using braces below the respective terms on the RHS of market clearance conditions. The following sets of equations relating supply (LHS) to demands (RHS) are part of the definition of an equilibrium:

Market for domestic output ( $p_{r,s}$ ):

$$Y_{s,r} \left( \frac{p_{s,r}}{py_{s,r}} \right)^\eta = A_{s,r} \underbrace{\left( \frac{pa_{s,r}}{p_{s,r}} \right)^{\sigma_a}}_{=dmi_{s,r}}. \quad (12)$$

Market for Armington aggregation ( $pa_{r,s}$ ):

$$\begin{aligned} \bar{a}_{s,r} A_{s,r} &= \sum_g \underbrace{\overline{vdifm}_{g,s,r}}_{=ddifm_{g,s,r}} Y_{s,r} \\ &+ \sum_h \underbrace{\left( \overline{vdpm}_{s,h,r} + \sum_{trd} \overline{vipm}_{s,trd,h,r} \frac{pc_{h,r}}{pa_{s,r}} \right)}_{=ddpm_{s,h,r} + dipm_{s,trd,h,r}} C_{h,r} \\ &+ \underbrace{\overline{vinvd}_{s,r}}_{=dinvd_{s,r}} INV_r \\ &+ \sum_{pub} \underbrace{\left( \overline{vdgm}_{s,r,pub} + \sum_{trd} \overline{vigm}_{s,trd,pub} \right)}_{=ddgm_{s,pub,r} + digm_{s,trd,pub,r}} GOV_{pub,r}. \end{aligned} \quad (13)$$

Market for intra-national trade ( $pn_s$ ):

$$\sum_r \overline{vxm}_{s,dtrd,r} Y_{s,r} \left( \frac{pn_s}{py_{s,r}} \right)^\eta = \sum_r A_{s,r} \underbrace{\overline{vim}_{s,dtrd,r} \left( \frac{pa_{s,r}}{cfn_{s,r}} \right)^{\sigma_a} \left( \frac{cfn_{s,r}}{pn_s} \right)^{\sigma_r}}_{=dim_{s,dtrd,r}}. \quad (14)$$

Market for investment ( $pinv_r$ ):

$$\overline{vinv}_r INV_r = \sum_h \overline{vinvh}_{h,r}. \quad (15)$$

Market for public consumption ( $pgov_{pub,r}$ ):

$$\overline{vgm}_{pub,r} GOV_{pub,r} pgov_{pub,r} = GOVT_{pub,r}. \quad (16)$$

Market for primary factors ( $pf_{fa,r}$ ):

$$\sum_h evo_{h,fa,r} = \sum_s \underbrace{\overline{vfm}_{fa,s,r}}_{=dfm_{s,fa,r}} Y_{s,r} \frac{cf_{s,r}}{pf_{fa,r}}. \quad (17)$$



Market for foreign exchange (*px*):

$$\begin{aligned} \sum_r \sum_h \overline{incadj}_{h,r} + \sum_{pub,r} \overline{vgm}_{pub,r} + \sum_r \sum_s \overline{uxm}_{s,ftd,r} Y_{s,r} \left( \frac{px}{py_{s,r}} \right)^\eta \\ = \sum_r \sum_s dfx_{s,r} + \sum_r \frac{TAXREV_r}{px}, \end{aligned} \quad (18)$$

where

$$dfx_{s,r} = A_{s,r} \underbrace{\overline{vim}_{s,ftd,r} \left( \frac{pa_{s,r}}{cfn_{s,r}} \right)^{\sigma_a} \left( \frac{cfn_{s,r}}{px} \right)^{\sigma_t}}_{=dim_{s,ftd,r}}.$$

Market for private consumption (*pc<sub>r,h</sub>*):

$$\overline{vpm}_{h,r} C_{h,r} pc_{h,r} = rh_{h,r}. \quad (19)$$

Market for price of carbon (*pcarb*):

$$\sum_r \sum_{pub} \overline{carbontarget}_{pub,r} = \sum_r \sum_s \overline{emit}_{s,r} Y_{s,r}. \quad (20)$$

Market for business taxes (*ptax*):

$$\sum_s \overline{vfm}_{btax,s,r} = \sum_s \overline{vfm}_{btax,s,r} Y_{s,r}. \quad (21)$$

#### 4.2.3 Income definitions

Private income (*RH<sub>h,r</sub>*):

$$RH_{h,r} = \sum_{fa} pf_{fa,r} \overline{evoh}_{h,fa,r} + px \overline{incadj}_{h,r} + pinv_r (-\overline{vinvh}_{h,r}). \quad (22)$$

Public income (*GOVT<sub>pub,r</sub>*):

$$GOVT_{pub,r} = px \overline{vgm}_{pub,r} + pcarb \overline{carbontarget}_{r,pub}. \quad (23)$$

Income of tax revenue agent (*TAXREV<sub>r</sub>*):

$$TAXREV_r = ptax_r \sum_s \overline{vfm}_{btax,s,r}. \quad (24)$$

This completes the sets of equations that describe the equilibrium conditions. The general equilibrium is defined by equations (7)-(24).

### 4.3 An Illustrative Counterfactual Experiment

In this section we calculate the economic costs of reducing the amount of emission permits in the economy. We assume that emission permits are issued by the federal government agent in each region. For simplicity, we assume that baseline emission by sector and region are given by:  $emit_{r,s} = 0.0001 Y_{r,s}$ . Emission permits can be traded economywide. In the initial equilibrium, emission permits are equal to total emissions commanding a price for carbon that equals zero.

Now consider the situation in which emission permits in each region are cut by  $x\%$ . In general, fewer emission permits drive up the price for carbon which in turn increases costs of production. Overall, this causes national and regional output, and welfare, measured as the change in Hicksian equivalent variation, to decrease. The virtue of carrying out this analysis in an AGE model which is calibrated on IMPLAN data, however, lies in the capability of disentangling economic costs by region and household type —as shown in Table 5.

TABLE 5: ECONOMIC IMPACTS OF A REDUCTION IN EMISSION PERMITS

| <i>Reduction in benchm. emission permits</i>  | 2 %   | 4 %   | 6 %    | 8 %    |
|---|-------|-------|--------|--------|
| <i>Change in national output (in %)</i>   | 0.30  | 1.05  | -2.10  | -6.38  |
| <i>Change in output by region (in %)</i>  |       |       |        |        |
| California  | 1.13  | 2.95  | 0.22   | -4.03  |
| Eastern and southern states   | 0.37  | 1.25  | -1.94  | -6.42  |
| Great Lakes states  | 0.07  | 0.44  | -2.43  | -6.06  |
| Plain states  | -0.25 | -0.22 | -3.77  | -8.19  |
| Western states (excluding CA)   | 0.15  | 0.91  | -2.33  | -6.58  |
| <i>Change in welfare by household (in %)</i><br>(results shown only for California) |       |       |        |        |
| CA.hh1  | -1.75 | -3.97 | -4.93  | -5.32  |
| CA.hh5  | -1.58 | -3.55 | -8.28  | -14.17 |
| CA.hh10   | -2.10 | -4.63 | -13.56 | -25.20 |
| CA.hh15   | -3.43 | -7.48 | -19.43 | -34.75 |
| CA.hh20   | -1.12 | -2.45 | -14.62 | -31.41 |
| CA.hh30   | -1.85 | -3.95 | -22.46 | -48.07 |
| CA.hh40   | -1.73 | -3.71 | -20.42 | -43.65 |
| CA.hh50   | -1.59 | -3.36 | -22.64 | -49.63 |
| CA.hh70   | -2.25 | -4.76 | -27.18 | -58.40 |

## 5 Practicalities

This section describes the usage of the IMPLANinGAMS package which can be downloaded at [www.mpsge.org](http://www.mpsge.org). The package is distributed as a zip file `IMPLAN2006inGAMS.zip` containing the directory structure and GAMS programs which unzips into a new directory.

### 5.1 System requirements

You will need to have the following:

- A recent GAMS system
- The PATH solver (installed GAMS subsystems)
- MPSGE subsystem
- Make sure you have the following files in the `inclib` subdirectory of your GAMS system: `domain.gms` and `unzip.gms`. If not download [www.mpsge.org/inclib/inclib.pck](http://www.mpsge.org/inclib/inclib.pck) into your GAMS system directory, and run `gamsinst`.

### 5.2 Getting started

GAMS source code for dataset and model management, and some template applications are provided with the distribution directory. The package is designed to operate with a complete national package, consisting of social accounting matrices for each of the 50 states. The IMPLAN datafiles are not included in the tools archive. They are not distributed freely and may be obtained from Minnesota IMPLAN Group (see [www.implan.com](http://www.implan.com)). Here are the steps involved in installing IMPLAN2006inGAMS:

1. Create an empty root directory named, say, `IMPLAN2006inGAMS`.
2. Unzip `IMPLAN2006inGAMS.zip` in the root directory. The root directory should have the following subdirectories:
  - **build** This directory contains a number of GAMS programs for reading IMPLAN datafiles, and for filtering, balancing and aggregation of the dataset.

- **data** This directory intends to hold all data (except for the original IMPLAN data files) in the GAMS Data eXchange (GDX) format. A number of subdirectories are created during different stages of dataset management process to hold temporary and final datasets.
- **defines** This directory contains set and mapping files which define model aggregation with respect to regions, sectors, factors of production and institutions. When aggregating a dataset, a `.set` file defines target sets and a `.map` file defines the mapping from source to target sets.
- **implandata** The source IMPLAN data files have to be placed in this directory. These can be individually zipped or flat `.gms` files. To be compatible with the programs provided here, the IMPLAN data files should be named in the following way: `st%year%-%state%.*`, where `%year%` stands for the last two digits of the base year (e.g., “06” if 2006) and `%state%` stands for the standard two-character abbreviation for states. We have appended the year of the IMPLAN source data in order to permit comparison of model results across alternative base year datasets.
- **listings** This directory holds all listing `.lst` files that are generated during the data management process.
- **models** This directory contains model source code and a few template applications.

3. Place the source state data files in the **implandata** directory and run `build.bat`.
4. Solve the sample models. The steps involved in testing the installation are in the **model** subdirectory. See `run.bat`.

### 5.3 Directory Contents

Here is an overview of the files provided in the IMPLAN2006inGAMS distribution:

#### ROOT DIRECTORY

- **build.bat** Master batch file controlling the conversion of IMPLAN state-level data files into GAMS-compatible data files. This program calls `readstate.bat` and `merge.gms`

- **readstate.bat** Batch file called by `build.bat` to read single state files and translate data into GTAP-style name space. In its default operation, this program calls `readall.gms` and `trns18.gms`. Optionally, the model `implan_acc.gms` may be run to verify the accounting identities implicit in the IMPLAN dataset.
- **run.bat** Batch file to run aggregation routine, balance domestic trade flows and verify benchmark consistency with a template model. In its default operation, this program calls `aggregation.gms`, `tradeadj.gms` and `soe_mcp`.

## BUILD

- **readall.gms** Reads a single IMPLAN state data file and stores the data in a `.gdx` file. In its default operation, this program diagonalizes IMPLAN data by converting the production structure into a commodity rather than industry basis to produce a more compact and numerically stable GE model. This feature can be deactivated by setting `.`. In addition, this program uses the GAMS routine `domain.gms` for efficient domain extraction.
- **trns18.gms** The purpose of this program is (i) to translate the IMPLAN data into GTAP-style data structures<sup>8</sup> and (ii) to filter the data and recalibrate the resulting dataset. The filter tolerance can be set by the parameter `tol`.
- **merge.gms** Merges single state data files into one file.
- **aggregation.gms** Aggregates an IMPLAN dataset.
- **tradeadj.gms** Performs adjustments of trade flows so that intra-state exports and imports balance.

## MODELS

- **implan\_acc.gms** A static state-level model that serves to illustrate the accounting identities implicit in the IMPLAN dataset.
- **soe\_mcp.gms** A prototype SOE regional US model to verify benchmark consistency. This version of the model is formulated in GAMS/MCP.

---

<sup>8</sup> Whenever possible, parameters holding benchmark data are chosen in accordance with the GTAP name space as in [Paltsev and Rutherford \[2000\]](#).

- **soe\_mge.gms** A prototype SOE regional US model to verify benchmark consistency. This version of the model is formulated in GAMS/MPSGE.

#### 5.4 Constructed data files

Here is an overview of the constructed data files:

- **data\noaggr\merged.gdx** Generated by `merge.gms`. Contains the national dataset consisting of all the state data files. This file holds data that has not been aggregated, i.e. contains individual data for all states, sectors, factors of production and institutions.
- **data\%target%\%target%.gdx** Generated by `aggregation.gms`. Contains a national aggregated dataset according to a mapping **target**. Domestic trade flows are not balanced.
- **data\%target%\%target%\_dtrdbal.gdx** Generated by `tradeadj.gms`. Contains a national aggregated dataset in which domestic trade flows have been balanced.

#### 5.5 Aggregation

The GAMS routines provided here are designed to permit flexibility with respect to the aggregation of regions, industries, factors of production and institutions. This is desirable because (i) it allows the modeler to easily generate a dataset that is suitable for his specific research question and (ii) careful model development typically requires to work with smaller datasets which facilitate debugging.

Aggregation of a dataset involves the following steps:

1. Target sets and mappings have to be defined in `.set` and `.map` files, respectively. Figure 7 provides an example of a `.set` file. Figure 8 displays a sample `.map` file defining regional and sectoral aggregation. Both files have to be named identically.
2. These files have to be placed in the **defines** subdirectory.
3. To invoke the defined sets and mappings, specify the target in the batch file `run.bat` using the name of the `.set` and `.map` file.
4. Run `run.bat` to aggregate the dataset (and to balance domestic trade flows and to verify benchmark consistency).

FIGURE 7: A SAMPLE .set FILE

```

SET reg Aggregate regions /
us /;

SET S Aggregated SAM accounts /

* 5 sectors + 5 energy

SRV Services
TRN Transportation
MAN Manufactured and Processed Goods
EIS Energy Intensive Sector
AGR Agriculture
GAS Natural Gas Distribution
COL Coal -- SIC 12
CRU Natural Gas and Crude -- SIC 13
ELE Electric generation -- SIC 49
OIL Refined Petroleum -- SIC 29/;

```

FIGURE 8: A SAMPLE .map FILE

```

SET mapr(*,*) Mapping from aggregate regions to states/

US.(AK,AL,AR,AZ,CA,CO,CT,DC,DE,FL,
GA,HI,IA,ID,IL,IN,KS,KY,LA,MA,
MD,ME,MI,MN,MO,MS,MT,NC,ND,NE,
NH,NJ,NM,NV,NY,OH,OK,OR,PA,RI,
SC,SD,TN,TX,UT,VA,VT,WA,WI,WV,
WY) /;

SET maps(*,*) Mapping from GTAP sectors to IMPLAN 509 /

OIL.( ! Petroleum, coal products
142 ! Petroleum refineries (324110:32411)
143 ! Asphalt paving mixture and block manufacturing (324121:324121)
144 ! Asphalt shingle and coating materials manufacturing (324122:324122)
145 ! Petroleum lubricating oil and grease manufacturing (324191:324191)
146 ! All other petroleum and coal products manufacturing (324199:324199)
),
CRU.( ! Crude oil and natural gas
19 ! Oil and gas extraction (211000:211)
...

```

## 5.6 Filtering of Small Values

The IMPLAN source data presents substantial challenges for calibrated models processed using direct solution methods (e.g., PATH, CONOPT or MINOS). Filtering is important because the presence of large numbers of small coefficients in the source data can cause numerical problems. These coefficients portray economic flows which are a negligible share of overall economic activity, yet they impose a significant computational burden during matrix factorization.

IMPLANinGAMS includes a piece of GAMS code (contained in **trns18.gms**) which removes small values and recalibrates the resulting dataset. An input to this program, the parameter `tol`, determines the filter tolerance. Values of TOL would normally range from  $1e-5$  to 0.01. Smaller values of `tol` retain a larger number of small coefficients in the filtered dataset.

Rounding to zero depends on relative tolerances as is shown here:

```
*      Drop sectors which have a negligible share of GDP:

zerotol(stol) = gdp*tol(stol)/10;
output(s,mkt)$ (abs(output(s,mkt)) < zerotol(stol)) = 0;

*      Drop inputs to production which are a negligible share of cost:

use(g,t,s)$ (use(g,t,s) < tol(stol)*sum(mkt,output(g,mkt))) = 0;
iuse(g,t,i)$ (iuse(g,t,i) < tol(stol)*sum(mkt,output(g,mkt))) = 0;
fd(f,t,s)$ (fd(f,t,s) < tol(stol)*sum(mkt,output(s,mkt))) = 0;
simpport(trd,t,s)$ (abs(simpport(trd,t,s)) < tol(stol)*sum(mkt,output(s,mkt))) = 0;

*      Drop institutional output which is negligibly small relative to
*      sectoral output:

imake(i,t,g)$ (abs(imake(i,t,g)) < tol(stol)*output(g,"dmkt")) = 0;

*      Drop factor endowments which are small relative to total supply:

fs(i,t,f)$ (abs(fs(i,t,f)) < tol(stol)*sum((tt,ff),fs(i,tt,ff))) = 0;
fexprt(f,t,trd)$ (abs(fexprt(f,t,trd)) < tol(stol)*sum((i,tt),fs(i,tt,f))) = 0;
fimpert(trd,t,f)$ (abs(fimpert(trd,t,f)) < tol(stol)*sum((i,tt),fs(i,tt,f))) = 0;
```



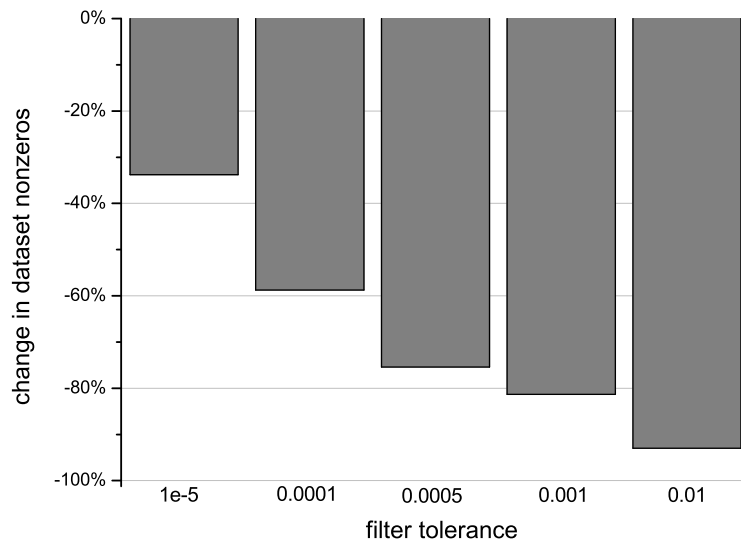


FIGURE 9: OVERALL IMPLAN FILTERING IMPACTS

Filtering makes a IMPLAN database smaller, as illustrated in the following Figure 9. As indicated, filtering reduces the size of a IMPLAN database by somewhere between 30% and 90%, depending on the filtering tolerance. The filtering procedure has differential impacts on various sub-matrices of the database. The largest proportional reduction in parameter density occurs in the *use* and *simport* (sectoral imports) matrices. There are also substantial reductions in the density of *output* and *fimport* (factor imports) arrays, as is indicated by Figure 10. Lastly, Figure 11 reveals that most of the reduction in nonzeros results from the elimination of small intermediate inputs and sectoral imports.

We believe that researchers will have their own opinions about how to select parameter values that should be rounded to zero. The discussion presented in this section intends to emphasize the more general point that the choice of filter tolerance does have a significant impact on the dataset development process.

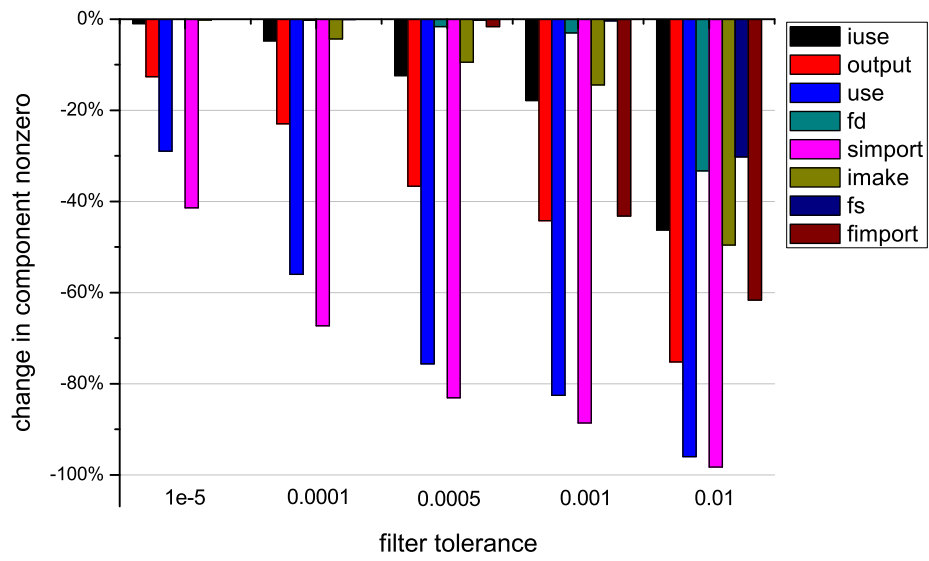


FIGURE 10: COMPONENT FILTERING RESULTS

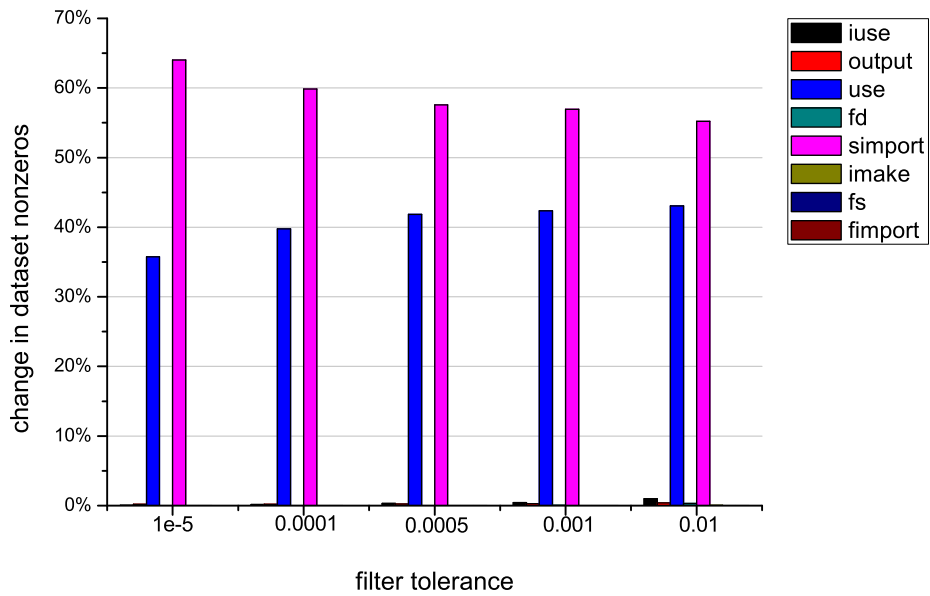


FIGURE 11: COMPONENT CONTRIBUTIONS

## References

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- PALTSEV, S. V., AND T. F. RUTHERFORD (2000): “GTAPinGAMS and GTAP-EG: Global Datasets for Economic Research and Illustrative Models,” Working Paper, Department of Economics, University of Colorado.
- RUTHERFORD, T. F. (1995): “Extension of GAMS for complementarity problems arising in applied economics,” *Journal of Economic Dynamics and Control*, 19(8), 1299–1324.
- (1999): “Applied general equilibrium modeling with MPSGE as a GAMS subsystem: an overview of the modeling framework and syntax,” *Computational Economics*, 14, 1–46.

## A GAMS/MPSGE Code: IMPLAN State-Level Accounting Model

\$title Simple Static State-Level Model

\* This model "implan\_acc.gms" serves to illustrate the accounting  
\* identities implicit in the IMPLAN dataset.

\$if not set ds \$set ds ak

\$batinclude statedata

\$ontext

\$model:soe

\$sectors:

|                      |                       |
|----------------------|-----------------------|
| y(s)\$vom(s)         | ! Sectoral production |
| m(g,trd)\$vim(g,trd) | ! Commodity imports   |
| cd(h)\$vpm(h)        | ! Household demand    |
| gd(pub)\$vgm(pub)    | ! Government demand   |
| id                   | ! Investment demand   |

\$commodities:

|                       |                     |
|-----------------------|---------------------|
| py(g)\$vom(g)         | ! Domestic output   |
| pm(g,trd)\$vim(g,trd) | ! Import price      |
| pc(h)\$vpm(h)         | ! Household demand  |
| pi                    | ! Investment        |
| pg(pub)               | ! Public demand     |
| pr(f)                 | ! Factor supplies   |
| prf                   | ! Corporate profits |
| pfx                   | ! Foreign exchange  |

\$consumers:

|         |                                      |
|---------|--------------------------------------|
| rh(h)   | ! Representative households          |
| rg(pub) | ! Representative public institutions |
| firms   | ! Representative firms               |

\$auxiliary:

|             |                             |
|-------------|-----------------------------|
| x(s)\$vx(s) | ! Sectoral exports          |
| fx          | ! Foreign exchange earnings |

\$prod:y(s)\$vom(s) s:0 t:1 va:1 g.tl:4  
o:py(s) q:vom(s)  
i:py(g) q:vdfm(g,s) g.tl:  
i:pm(g,trd) q:vifm(g,trd,s) g.tl:  
i:pr(f) q:vfm(f,s) va:

\$prod:m(g,trd)\$vim(g,trd)  
o:pm(g,trd) q:vim(g,trd)  
i:prfx q:vim(g,trd)

\$prod:cd(h)\$vpm(h) s:1 g.tl:4  
o:pc(h) q:vpm(h)  
i:py(g) q:vdpm(g,h) g.tl:  
i:pm(g,trd) q:vipm(g,trd,h) g.tl:

\$prod:id  
o:pi q:vinv  
i:py(g) q:vdim(g)  
i:pm(g,trd) q:viim(g,trd)

\$prod:gd(pub)\$vgm(pub)  
o:pg(pub) q:vgm(pub)  
i:py(g) q:vdgm(g, pub)  
i:pm(g, trd) q:vigm(g, trd, pub)

\$demand:rh(h)\$vpm(h)  
d:pc(h) q:vpm(h)  
e:prf q:vprf(h)  
e:pr(f) q:(sum(t, evom(f, h, t)))  
e:py(g) q:evpm(g, h)  
e:prfx q:(sum(t, vtrn(h, t)))

\$demand:rg(pub)\$vgm(pub)  
d:pg(pub) q:vgm(pub)  
e:prf q:vprf(pub)  
e:pr(f) q:(sum(t, evom(f, pub, t)))  
e:py(g) q:evpm(g, pub)  
e:prfx q:(sum(t, vtrn(pub, t)))

```

$demand:firms
    d:prf          q:(sum(i,vprf(i)))
    e:pi           q:(-vinv)
    e:pr(f)       q:(sum((corp,t),evom(f,corp,t)))
    e:py(g)       q:(sum(corp,evpm(g,corp)))
    e:pfx         q:(sum((corp,t),vtrn(corp,t)))
    e:py(s)$vx(s) q:(-1)  r:x(s)
    e:pfx         q:1     r:fx

```

```

$constraint:x(s)$vx(s)
    x(s) =e= sum(trd, vxm(s,trd) * (pfx/py(s))**5);

```

```

$constraint:fx
    fx * pfx =e= sum(s$vx(s), py(s)*x(s));

```

```

$offtext

```

```

$sysinclude mpsgeset soe

```

```

x.l(s) = vx(s);
fx.l = sum(s, vx(s));

```

```

soe.workspace = 25;
soe.iterlim = 0;
$include soe.gen
solve soe using mcp;

```

## B GAMS/MCP Code: Prototype US Regional Model

```
$title Simple Static Small Open Economy Model with Intra-national Trade
```

```
$if not set target $set target gtap
```

```
* Read the dataset using the utility program:
```

```
$batainclude models\regionaldata
```

```
* Define set for primary factor of production and taxes:
```

```
SET fa(f) /empl,prop,othp/  
ft(f) /btax/;
```

```
ALIAS (fa,ffa);
```

```
* Install parameters for model specification:
```

```
PARAMETER
```

```
vdifm(r,g,s) Total intermediate demand  
evo(r,i,f) Factor endowment by institution  
va(r,s) Armington supply including imports  
vn(g) Intra-national trade  
vinvd(r,g) Investment demand by commodity  
vinvh(r,h) Investment demand by household  
incadj(r,h) Base year net transfer  
tc(r,s) Carbon tax rate  
carbontarget Carbon emissions target  
emit(r,s) Base year emissions;
```

```
vdifm(r,g,s) = vdfm(r,g,s) + sum(trd, vifm(r,g,trd,s));
```

```
evo(r,i,f) = sum(t, evom(r,f,i,t));
```

```
va(r,s) = sum(trd,vim(r,s,trd)) + vdmi(r,s);
```

```
vn(g) = sum(r, vxm(r,g,"dtrd"))+eps;
```

```
* Impute investment demand to households:
```

```
vinvd(r,g) = va(r,g) - (sum(h,vdpm(r,g,h))+sum((trd,h),vipm(r,g,trd,h)))  
- (sum(pub,vdgm(r,g,pub))+sum((trd,pub),vigm(r,g,trd,pub)))
```

- sum(s, vdifm(r,g,s));

\* Impute wage payments, capital payments:

vfm(r,"empl",s) = max(vdmi(r,s)+sum(trd,vxm(r,s,trd))  
- (vfm(r,"prop",s) + vfm(r,"othp",s)) - vfm(r,"btax",s)  
- sum(g, vdifm(r,g,s)), 0);

vfm(r,"prop",s) = max(vdmi(r,s)+sum(trd,vxm(r,s,trd))  
- (vfm(r,"empl",s) + vfm(r,"othp",s)) - vfm(r,"btax",s)  
- sum(g, vdifm(r,g,s)), 0);

vfm(r,"othp",s) = max(vdmi(r,s)+sum(trd,vxm(r,s,trd))  
- (vfm(r,"empl",s) + vfm(r,"prop",s)) - vfm(r,"btax",s)  
- sum(g, vdifm(r,g,s)), 0);

vfm(r,"btax",s) = vdmi(r,s)+sum(trd,vxm(r,s,trd))  
- (vfm(r,"prop",s) + vfm(r,"othp",s)) - vfm(r,"empl",s)  
- sum(g, vdifm(r,g,s));

\* Scale labor endowments to match imputed wage payments:

evo(r,h,"empl") = evo(r,h,"empl")/sum(hh, evo(r,hh,"empl"))  
\* sum(s, vfm(r,"empl",s));

evo(r,h,"prop") = evo(r,h,"prop")/sum(hh,evo(r,hh,"prop"))  
\* sum(s, vfm(r,"prop",s));

evo(r,h,"othp") = evo(r,h,"othp")/sum(hh,evo(r,hh,"othp"))  
\* sum(s, vfm(r,"othp",s));

\* Impute aggregate investment and investment demand by household:

vinv(r) = sum(g, vinvd(r,g));  
vinvh(r,h)\$sum(hh, max(0, sum(t,transfer(r,"inv",t,hh) - transfer(r,hh,t,"inv"))))  
= max(0, sum(t, transfer(r,"inv",t,h) - transfer(r,h,t,"inv"))) /  
sum(hh, max(0, sum(t, transfer(r,"inv",t,hh)  
- transfer(r,hh,t,"inv"))))\* vinv(r);

\* Adjust household income:

incadj(r,h) = sum(g,vdpm(r,g,h)+sum(trd,vipm(r,g,trd,h))  
+ vinvh(r,h) - sum(f,evo(r,h,f)));

\* Base year emissions:



```
emit(r,s) = 1e-4 * (vdmi(r,s) + sum(trd,vxm(r,s,trd)));
carbontarget(r) = sum(s,emit(r,s));
```

\* Define elasticity parameters:

PARAMETER

```
eta      ES of transformation /2/
es_a     EOS bw domestic and traded goods /4/
es_t     EOS bw domestically and foreign traded goods /8/;
```

\* Define value share parameters to simplify algebra:

PARAMETER

```
thetai(r,s)  Value share of intermediate input
thetaf(r,s)  Value added share of sectoral output
thetae(r,s)  Value share of emmissions
thetat(r,s)  Value share of business taxes;
```

```
thetai(r,s)$vdmi(r,s) = sum(g,vdifm(r,g,s))/(vdmi(r,s)+sum(trd,vxm(r,s,trd)));
thetaf(r,s)$vdmi(r,s) = sum(fa,vfm(r,fa,s))/(vdmi(r,s)+sum(trd,vxm(r,s,trd)));
thetae(r,s)$vdmi(r,s) = emit(r,s)/(vdmi(r,s)+sum(trd,vxm(r,s,trd)));
thetat(r,s)$vdmi(r,s) = vfm(r,"btax",s)/(vdmi(r,s)+sum(trd,vxm(r,s,trd)));
```

\*-----

\* Formulate a stylized SOE US regional model in GAMS/MCP based on  
 \* hese statistics and verify that the resulting data represents  
 \* an equilibrium:

POSITIVE VARIABLES

```
y(r,s)      Sectoral production
a(r,s)      Armington aggregation
c(r,h)      Consumption by household
gov(r,pub)  Public output
inv(r)      Investment

p(r,s)      Sectoral output prices
pa(r,s)     Armington aggregate prices
pc(r,h)     Consumption by household
pn(s)      Intra-national trade price
```

|               |  |
|---------------|--|
| pinv(r)       | New investment                                   |
| pgov(r, pub)  | Public output                                    |
| pf(r, fa)     | Factor prices                                    |
| px            | Foreign exchange                                 |
| pcarb         | Tradable CO2 emission permit price               |
| ptax(r)       | Business taxes                                   |
|               |  |
| rh(r, h)      | Representative households                        |
| govt(r, pub)  | Government (different levels)                    |
| taxrev        | Tax revenue agent                                |
|               |  |
| target        | Rationing variable for emission permits          |
|               |  |
| cf(r, s)      | User cost index for primary factors              |
| ca(r, s)      | User cost index for Armington inputs             |
| cfn(r, g)     | User cost index for domestic and imported inputs |
| dfm(fa, r, s) | Sectoral demand for primary factors              |
| dfx(r, g)     | Armington demand for foreign exchange            |
| dfn(r, g)     | Armington demand for domestic trade              |
| da(r, s, g)   | Sectoral demand for Armington good               |
| dad(r, g)     | Armington demand for domestic output             |
| py(r, s)      | Price for sectoral output;                       |

#### EQUATIONS

|                  |                                    |
|------------------|------------------------------------|
| prf_y(r, s)      | Sectoral production                |
| prf_a(r, s)      | Armington aggregation              |
| prf_c(r, h)      | Consumption by household           |
| prf_gov(r, pub)  | Public output                      |
| prf_inv(r)       | Investment                         |
|                  |                                    |
| mkt_p(r, s)      | Sectoral output prices             |
| mkt_pa(r, s)     | Armington aggregate prices         |
| mkt_pc(r, h)     | Consumption by household           |
| mkt_pn(s)        | Intra-national trade price         |
| mkt_pinv(r)      | New investment                     |
| mkt_pgov(r, pub) | Public output                      |
| mkt_pf(r, fa)    | Factor prices                      |
| mkt_pfx          | Foreign exchange                   |
| mkt_pcarb        | Tradable CO2 emission permit price |
| mkt_ptax(r)      | Business taxes                     |

inc\_rh(r,h) Representative households  
 inc\_govt(r,pub) Government (different levels)  
 inc\_taxrev(r) Tax revenue agent

eq\_cf(r,s)  
 eq\_ca(r,s) User cost index for Armington inputs  
 eq\_cfn(r,g) User cost index for domestic and imported inputs  
 eq\_dfm(fa,r,s) Sectoral demand for primary factors  
 eq\_dfx(r,g) Armington demand for foreign exchange  
 eq\_dfn(r,g) Armington demand for domestic trade  
 eq\_da(r,s,g) Sectoral demand for Armington good  
 eq\_dad(r,g) Armington demand for domestic output  
 eq\_py(r,s) Price for sectoral output;

\* Equation definitions to simplify algebra:

eq\_cf(r,s) $\$(\text{sum}(fa, \text{vfm}(r, fa, s)))$ .. cf(r,s) =e= prod(fa, pf(r,fa)\*\*(vfm(r,fa,s)  
 /sum(ffa,vfm(r,ffa,s))));

eq\_ca(r,s) $\$(\text{sum}(g, \text{vdifm}(r, g, s)))$ .. ca(r,s) =e=  
 sum(g, vdifm(r,g,s)/sum(gg,vdifm(r,gg,s)) \* PA(r,g));

eq\_cfn(r,g) $\$(\text{sum}(\text{trd}, \text{vim}(r, g, \text{trd})))$ .. cfn(r,g) =e=  
 (vim(r,g,"ftrd")/sum(trd,vim(r,g,trd))\*PFX\*\*(1-es\_t)  
 + vim(r,g,"dtrd")/sum(trd,vim(r,g,trd))  
 \*PN(g)\*\*(1-es\_t)\*\*(1/(1-es\_t)));

eq\_dfm(fa,r,s) .. dfm(fa,r,s) =e= vfm(r,fa,s)\*Y(r,s)\*cf(r,s)/pf(r,fa);

eq\_dfx(r,g) .. dfx(r,g) =e= A(r,g) \* vim(r,g,"ftrd")  
 \*(PA(r,g)/cfn(r,g))\*\*es\_a \* (cfn(r,g)/PFX)\*\*es\_t;

eq\_dfn(r,g) .. dfn(r,g) =e= A(r,g) \* vim(r,g,"dtrd")  
 \*(cfn(r,g)/PA(r,g))\*\*(-es\_a) \* (cfn(r,g)/PN(g))\*\*es\_t;

eq\_da(r,s,g) .. da(r,s,g) =e= vdifm(r,g,s)\*Y(r,s);

eq\_dad(r,g) .. dad(r,g) =e= vdmi(r,g)\*A(r,g)\*(PA(r,g)/P(r,g))\*\*es\_a;

```

eq_py(r,s)$(vdmi(r,s)+sum(trd,vxm(r,s,trd))).. py(r,s) =e=
    (vdmi(r,s)/(vdmi(r,s)+sum(trd,vxm(r,s,trd)))*P(r,s)**(1+eta)
    + vxm(r,s,"ftrd")/(vdmi(r,s)+sum(trd,vxm(r,s,trd)))*PFX**(1+eta)
    + vxm(r,s,"dtrd")/(vdmi(r,s)+sum(trd,vxm(r,s,trd)))*PN(s)**(1+eta)
    )**(1/(1+eta));

```

\* Zero profit conditions:

```

prf_y(r,s)$(vdmi(r,s)+sum(trd,vxm(r,s,trd)))
    .. thetai(r,s)*ca(r,s) + thetaf(r,s)*cf(r,s) + thetae(r,s)*PCARB
    + thetat(r,s)*PTAX(r) =e= py(r,s);

```

```

prf_a(r,s)$va(r,s).. (vdmi(r,s)/va(r,s)*(P(r,s)**(1-es_a)) + sum(trd,vim(r,s,trd))
    /va(r,s)*cfn(r,s)**(1-es_a)**(1/(1-es_a)) =e= PA(r,s);

```

```

prf_inv(r).. sum(g, PA(r,g) * vinvd(r,g)) =e= PINV(r) * vinv(r);

```

```

prf_gov(r,pub).. sum(g, PA(r,g) * (vdgm(r,g,pub)+sum(trd,vigm(r,g,trd,pub))))
    =e= PGOV(r,pub) * vgm(r,pub);

```

```

prf_c(r,h).. pc(r,h) =e= prod(s, pa(r,s)**((vdpm(r,s,h)
    + sum(trd,vipm(r,s,trd,h)))
    /sum(ss,(vdpm(r,ss,h) + sum(trd,vipm(r,ss,trd,h))))));

```

\* Market clearing conditions:

```

mkt_p(r,s)$(vdmi(r,s)+sum(trd,vxm(r,s,trd))).. vdmi(r,s)*Y(r,s)*(P(r,s)/PY(r,s))**eta
    =e= dad(r,s);

```

```

mkt_pa(r,s)$va(r,s).. va(r,s)*A(r,s) =e= sum(g, da(r,g,s))
    + sum(h, (vdpm(r,s,h)+sum(trd,vipm(r,s,trd,h)))*PC(r,h)/PA(r,s)*C(r,h))
    + vinvd(r,s)*INV(r)
    + sum(pub, (vdgm(r,s,pub)+sum(trd,vigm(r,s,trd,pub)))*GOV(r,pub));

```

```

mkt_pn(s).. sum(r, vxm(r,s,"dtrd")*Y(r,s)*(PN(s)/PY(r,s))**eta)
    =e= sum(r, dfn(r,s));

```

```

mkt_pinv(r).. vinv(r) * INV(r) =e= sum(h, vinvh(r,h));

```

mkt\_pgov(r, pub).. vgm(r, pub) \* GOV(r, pub) \* PGOV(r, pub) =e= GOVT(r, pub);

mkt\_pf(r, fa).. sum(h, evo(r, h, fa)) =e= sum(s, dfm(fa, r, s));

mkt\_pfx.. sum((r, h), incadj(r, h)) + sum((r, pub), vgm(r, pub))  
+ sum((r, s), vxm(r, s, "ftrd")\*Y(r, s)\*(PFX/PY(r, s))\*\*eta)  
=e= sum((r, s), dfx(r, s)) + sum(r, taxrev(r)/PFX);

mkt\_pc(r, h).. vpm(r, h) \* C(r, h) \* PC(r, h) =e= RH(r, h);

mkt\_pcarb.. sum((r, pub), carbontarget(r)\$ (ord(pub) eq 1)) =e= sum((r, s), emit(r, s) \* Y(r, s));

mkt\_ptax(r).. sum(s, vfm(r, "btax", s)) =e= sum(s, vfm(r, "btax", s) \* Y(r, s));

\* Income definitions:

inc\_rh(r, h).. RH(r, h) =e= sum(fa, pf(r, fa)\*evo(r, h, fa))  
+ pfx\*incadj(r, h) + pinv(r)\*(-vinvh(r, h));

inc\_govt(r, pub).. GOVT(r, pub) =e= pfx\*vgm(r, pub) + pcarb\*carbontarget(r)\$ (ord(pub) eq 1);

inc\_taxrev(r).. TAXREV(r) =e= ptax(r) \* sum(s, vfm(r, "btax", s));

\* Define model and match equations and variables:

model soe\_mcp /prf\_y.y,prf\_a.a,prf\_c.c,prf\_gov.gov,prf\_inv.inv,mkt\_p.p,  
mkt\_pa.pa,mkt\_pc.pc,mkt\_pn.pn,mkt\_pinv.pinv,mkt\_pgov.pgov,  
mkt\_pf.pf,mkt\_pfx.pfx,mkt\_pcarb.pcarb,mkt\_ptax.ptax,inc\_rh.rh,  
inc\_govt.govt,inc\_taxrev.taxrev,eq\_cf.cf,  
eq\_ca.ca,eq\_cfn.cfn,eq\_dfm.dfm,eq\_dfx.dfx,eq\_dfn.dfn,eq\_da.da,  
eq\_dad.dad,eq\_py.py/;

\* Assign default values:

y.l(r, s)=1;a.l(r, s)=1;c.l(r, h)=1;gov.l(r, pub)=1;inv.l(r)=1;p.l(r, s)=1;  
pa.l(r, s)=1;pc.l(r, h)=1;pn.l(s)=1;pinv.l(r)=1;pgov.l(r, pub)=1;pf.l(r, fa)=1;  
pfx.l=1;pcarb.l=0;ptax.l(r)=1;rh.l(r, h)=vpm(r, h);govt.l(r, pub)=vgm(r, pub);  
cf.l(r, s)=1;ca.l(r, s)=1;cfn.l(r, g)=1;dfm.l(fa, r, s)=vfm(r, fa, s);  
dfx.l(r, g)=vim(r, g, "ftrd");dfn.l(r, g)=vim(r, g, "dtrd");da.l(r, s, g)=vdifm(r, g, s);  
dad.l(r, g)=vdm(r, g);py.l(r, s)=1;taxrev.l(r)=sum(s, vfm(r, "btax", s));

\* Fix variables which should not be in the model:

```
PN.fx(s)$(vn(s)=0) =1;
```

\* Choose a numeraire:

```
PFX.fx =1;
```

\* Verify benchmark consistency:

```
soe_mcp.iterlim = 0;
```

```
solve soe_mcp using mcp;
```

## C GAMS/MPSGE Code: Prototype US Regional Model

```
$title Simple Static Small Open Economy Model with Intra-national Trade
```

```
$if not set target $set target gtap
```

```
* Read the dataset using the utility program:
```

```
$batinclude models\regionaldata
```

```
* Define set for primary factor of production and taxes:
```

```
SET fa(f) /empl,prop,othp/  
ft(f) /btax/;
```

```
* Install parameters for model specification:
```

```
PARAMETER
```

|              |                                    |
|--------------|------------------------------------|
| vdifm(r,g,s) | Total intermediate demand          |
| evo(r,i,f)   | Factor endowment by institution    |
| va(r,s)      | Armington supply including imports |
| vn(g)        | Intra-national trade               |
| vinvd(r,g)   | Investment demand by commodity     |
| vinvh(r,h)   | Investment demand by household     |
| incadj(r,h)  | Base year net transfer             |
| tc(r,s)      | Carbon tax rate                    |
| carbontarget | Carbon emissions target            |
| emit(r,s)    | Base year emissions;               |

```
vdifm(r,g,s) = vdfm(r,g,s) + sum(trd, vifm(r,g,trd,s));
```

```
evo(r,i,f) = sum(t, evom(r,f,i,t));
```

```
va(r,s) = sum(trd,vim(r,s,trd)) + vdmi(r,s);
```

```
vn(g) = sum(r, vxm(r,g,"dtrd"));
```

```
* Impute investment demand to households:
```

```
vinvd(r,g) = va(r,g) - (sum(h,vdpm(r,g,h))+sum((trd,h),vipm(r,g,trd,h)))  
- (sum(pub,vdgm(r,g,pub))+sum((trd,pub),vigm(r,g,trd,pub)))  
- sum(s, vdifm(r,g,s));
```

\* Impute wage payments, capital payments:

```

vfm(r,"empl",s) = max(vdmi(r,s)+sum(trd,vxm(r,s,trd))
                    - (vfm(r,"prop",s) + vfm(r,"othp",s)) - vfm(r,"btax",s)
                    - sum(g, vdifm(r,g,s)), 0);
vfm(r,"prop",s) = max(vdmi(r,s)+sum(trd,vxm(r,s,trd))
                    - (vfm(r,"empl",s) + vfm(r,"othp",s)) - vfm(r,"btax",s)
                    - sum(g, vdifm(r,g,s)), 0);
vfm(r,"othp",s) = max(vdmi(r,s)+sum(trd,vxm(r,s,trd))
                    - (vfm(r,"empl",s) + vfm(r,"prop",s)) - vfm(r,"btax",s)
                    - sum(g, vdifm(r,g,s)), 0);
vfm(r,"btax",s) = vdmi(r,s)+sum(trd,vxm(r,s,trd))
                    - (vfm(r,"prop",s) + vfm(r,"othp",s)) - vfm(r,"empl",s)
                    - sum(g, vdifm(r,g,s));

```

\* Scale labor endowments to match imputed wage payments:

```

evo(r,h,"empl") = evo(r,h,"empl")/sum(hh, evo(r,hh,"empl"))
                * sum(s, vfm(r,"empl",s));
evo(r,h,"prop") = evo(r,h,"prop")/sum(hh,evo(r,hh,"prop"))
                * sum(s, vfm(r,"prop",s));
evo(r,h,"othp") = evo(r,h,"othp")/sum(hh,evo(r,hh,"othp"))
                * sum(s, vfm(r,"othp",s));

```

\* Impute aggregate investment and investment demand by household:

```

vinv(r) = sum(g, vinvd(r,g));
vinvh(r,h)$sum(hh, max(0, sum(t,trnsfer(r,"inv",t,hh) - trnsfer(r,hh,t,"inv"))))
        = max(0, sum(t, trnsfer(r,"inv",t,h) - trnsfer(r,h,t,"inv"))) /
          sum(hh, max(0, sum(t, trnsfer(r,"inv",t,hh)
          - trnsfer(r,hh,t,"inv"))))* vinv(r);

```

\* Adjust household income:

```

incadj(r,h) = sum(g,vdpm(r,g,h)+sum(trd,vipm(r,g,trd,h)))
            + vinvh(r,h) - sum(f,evo(r,h,f));

```

\* Base year emissions:

```

emit(r,s) = 1e-4 * (vdmi(r,s) + sum(trd,vxm(r,s,trd)));

```



carbontarget(r) = sum(s,emit(r,s));

\* Define elasticity parameters:

PARAMETER

eta ES of transformation /2/

es\_a EOS bw domestic and traded goods /4/

es\_t EOS bw domestically and foreign traded goods /8/;

\*-----

\* Formulate a stylized SOE US regional model in GAMS/MPSGE based on  
\* these statistics and verify that the resulting data represents  
\* an equilibrium:

\$ontext

\$model:soe

\$sectors:

y(r,s)\$(vdmi(r,s)+sum(trd,vxm(r,s,trd))) ! Sectoral production

a(r,s)\$va(r,s) ! Armington aggregation

c(r,h) ! Consumption by household

gov(r,pub) ! Public output

inv(r) ! Investment

\$commodities:

p(r,s)\$(vdmi(r,s)+sum(trd,vxm(r,s,trd))) ! Sectoral output prices

pc(r,h) ! Consumption by household

pa(r,s)\$va(r,s) ! Armington aggregate prices

pn(s)\$vn(s) ! Intra-national trade price

pinv(r) ! New investment

pgov(r,pub) ! Public output

pf(r,fa)\$(sum(s,vfm(r,fa,s))) ! Factor prices

pfx ! Foreign exchange

pcarb ! Tradable CO2 emission permit price

ptax(r)\$(sum(s,vfm(r,"btax",s))) ! Business taxes

\$consumers:

rh(r,h) ! Representative households

govt(r,pub) ! Government (different levels)

taxrev(r) ! Tax revenue agent

\$prod:y(r,s)\$ (vdmi(r,s)+sum(trd,vxm(r,s,trd))) s:0 t:eta va:1

o:p(r,s) q:vdmi(r,s)  
o:px q:vxm(r,s,"ftrd")  
o:pn(s) q:vxm(r,s,"dtrd")  
i:pa(r,g) q:vdifm(r,g,s)  
i:pf(r,fa) q:vfm(r,fa,s) va:  
i:pcarb q:emit(r,s)  
i:ptax(r) q:vfm(r,"btax",s)

\$prod:a(r,s)\$va(r,s) s:es\_a m:es\_t

o:pa(r,s) q:va(r,s)  
i:p(r,s) q:vdmi(r,s)  
i:px q:vim(r,s,"ftrd") m:  
i:pn(s) q:vim(r,s,"dtrd") m:

\$prod:inv(r)

o:pinv(r) q:vinv(r)  
i:pa(r,s) q:vinvd(r,s)

\$prod:gov(r, pub)

o:pgov(r, pub) q:vgm(r, pub)  
i:pa(r, s) q:(vdgm(r, s, pub)+sum(trd, vigm(r, s, trd, pub)))

\$prod:c(r, h) s:1

o:pc(r, h) q:vpm(r, h)  
i:pa(r, s) q:(vdpm(r, s, h)+sum(trd, vipm(r, s, trd, h)))

\$demand:rh(r, h)

d:pc(r, h) q:vpm(r, h)  
e:pf(r, fa) q:evo(r, h, fa)  
e:px q:incadj(r, h)  
e:pinv(r) q:(-vinvh(r, h))

\$demand:govt(r, pub)

d:pgov(r, pub) q:vgm(r, pub)  
e:px q:vgm(r, pub)  
e:pcarb q:(carbontarget(r)\$ (ord(pub) eq 1))

```

$demand:taxrev(r)
      d:pfx
      e:ptax(r)      q:(sum(s,vfm(r,"btax",s)))

$report:
      v:yd(r,s)      o:p(r,s)      prod:y(r,s)
      v:yftrd(r,s)   o:pfx      prod:y(r,s)
      v:ydtrd(r,s)   o:pn(s)      prod:y(r,s)

$offtext
$sysinclude mpsgeset soe

*      Choose a numeraire:

PFX.fx = 1;

soe.workspace = 40;
soe.iterlim = 1;
$include soe.gen
solve soe using mcp;
abort$(abs(soe.objval) gt 1e-8) "***Model does not calibrate***";

*-----
*      Run a series of counterfactuals which reduce the amount of emission permits
*      in the economy:

*      Define report parameters:

PARAMETER      rcarb      Percentage reduction of base year emission (permits)
               rpcarb      Price of carbon
               cons      Welfare change by household tpe and region
               no_b      Benchmark value of national output
               ro_b      Benchmark value of regional output
               nationaloutput Change in national output (in%)
               regionaloutput Change in regional output (in%);

no_b = sum((r,s),yd.l(r,s)*p.l(r,s) + yftrd.l(r,s)*pfx.l+ydtrd.l(r,s)*pn.l(s));
ro_b(r) = sum(s,yd.l(r,s)*p.l(r,s) + yftrd.l(r,s)*pfx.l+ydtrd.l(r,s)*pn.l(s));

*      Define an iteration index:

```

```

SET z /1*4/;

*      Solve a series of models (in the benchmark the price for carbon is zero,
      ie the cap is not binding):

LOOP(z,

*      Reduce emission permits:

carbontarget(r) = (1-0.02*ord(z))*sum(s,emit(r,s));

soe.iterlim = 10000;
$include soe.gen
solve soe using mcp;

rcarbt(z) = 100*ord(z)*0.02;
cons("CA",h,z) = 100*(c.l("CA",h)-1);
nationaloutput(z) = 100*(sum((r,s),yd.l(r,s)*p.l(r,s) + yftrd.l(r,s)
      *pfx.l+ydtrd.l(r,s)*pn.l(s))/no_b-1);
regionaloutput(r,z) = 100*(sum(s,yd.l(r,s)*p.l(r,s) + yftrd.l(r,s)
      *pfx.l+ydtrd.l(r,s)*pn.l(s))/ro_b(r)-1);
rpcarb(z) = pcarb.l;

);

DISPLAY rcarbt,rpcarb,cons,nationaloutput,regionaloutput;

```