GTAP7inGAMS

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Abstract

This paper describes the implementation of the GTAP7 model within the GAMS modeling language. Differences with previous implementations are discussed.

1 Introduction

The Global Trade Analysis Project (GTAP) is a research program initiated in 1992 to provide the economic research community with a global economic dataset for use in the quantitative analyses of international economic issues. The project's objectives include the provision of a documented, publicly available, global, general equilibrium data base, and to conduct seminars on a regular basis to inform the research community about how to use the data in applied economic analysis. GTAP has lead to the establishment of a global network of researchers who share a common interest of multi-region trade analysis and related issues. The GTAP research program is coordinated by Thomas Hertel, Director of the Center for Global Trade Analysis at Purdue University. As Deputy Director of this Center, Robert McDougall oversees the data base work. Software development within the GTAP project has been assisted greatly by the efforts of Ken Pearson, Mark Horridge and other Australian researchers from Centre of Policy Studies, Monash University. (See Hertel [1997] and McDougall [2005]). A list of applications based on the GTAP framework can be found at the GTAP home page, (HTTP://WWW.GTAP.ORG).

1.1 GTAP 7 Database

The GTAP version 7 database represents global production and trade for 113 country/regions, 57 commodities and 5 primary factors, two of which are "sluggish" (imper-

*The research assistance of Justin Carone is gratefully acknowledged. I remain responsible for all errors remaining in the code and documentation, of which there are undoubtedly many. I remain grateful for any bug fixes or suggestions for clarification of the documentation which may be offered by users of these tools.

fectly mobile across sectors). The data characterize intermediate demand and bilateral trade in 2004, including tax rates on imports and exports and other indirect taxes.

A guide to what's new in GTAP7 can be found in [Narayanan and Dimaranan 2008].

1.2 GAMS implementation

The principal programming language for GTAP data and modeling work is GEMPACK [Harrison and Pearson 1996]. In the GEMPACK framework the model is solved as a system of nonlinear equations. The present paper describes a version of the GTAP model which has been implemented in GAMS. The GAMS model is essentially implemented as a nonlinear system of equations, although it can be posed either as a CNS or MCP. Along with the core model I have developed two ancillary programs for dataset management. The package is called "GTAP7inGAMS". These programs should be useful to economists who program in GAMS and wish to use GTAP in applied work. These programs include tools for translation of the GTAP files into GAMS readable form, GAMS programs for dataset aggregation and reconciliation. These tools permit users to easily filter and adjust tax rates on trade or domestic transactions.

1.3 Differences with previous versions

This version of GTAPinGAMS differs in a couple of ways from the previous implementation (GTAP6inGAMS), although the differences are primarily technical or cosmetic.

GTAP7inGAMS implements a new nameset g which includes all produced goods i as well as private consumption, government expenditures, and investment. This allows to drop all accounting values specific to these last three components by treating them analogously to sectoral production. The presentation of the model is thus simplified without changing its underlying structure.

In this edition of the model a single consumer in each region collects domestic taxes and demands private, public and investment goods. This simplifies exposition of the model and removes the need for artificial transfers between households and government without fundamentally altering the model structure.

Furthermore, the code takes advantage of the `macro` function implemented in version 22.9 of GAMS[1], eliminating the need for the GAMS-F preprocessor, and making the algebraic version of the model more compact and readable.

1.4 Differences with the GEMPACK implementation of GTAP

There are a few substantive differences between the GEMPACK and GAMS version of the model. One of these is the final demand system. Whereas the GEMPACK model is based on a CDE demand system, the GAMS model employs Cobb-Douglas preferences.[2] Second, there are differences in units of account. Values in the GAMS implementation differ from the GEMPACK model by a factor of 1000. The GTAP database measures

[1] Bruce McCarl's GAMS User Guide and the GAMS Resease Notes for details. Version 23.1 of GAMS is required

[2] We include an alternative model based on a non-separable nested quasi-homothetic, CES which calibrates to second-order properties of the CDE model This LES-NNCES demand system matches the own-price and income elasticities of demand from GEMPACK model.

all transactions in millions of dollars whereas GTAP7inGAMS measures transactions in billions of dollars. Third, the two models differ in their representation of investment demand and global capital markets. The GEMPACK model assumes that a "global bank" allocates international capital flows in response to changes in regional rates of return. The GTAP7inGAMS model makes the simplest possible assumptions regarding investment demand, international capital flows and the time path of adjustment: all of these variables are exogenously fixed at base year levels.

1.5 Structure

This paper consists of three sections following this overview. Section 2 introduces the dataset and core static model. I have written this material with the hope that that the model might be accessible to a second year graduate student who has never worked with GTAP. The exposition begins with flow charts and stick diagrams, it then presents a "primal" version of the model with quantity variables and individual optimization problems. Thereafter, I conclude with a "dual" ([Dixit and Norman 1992]) version of the model which closely follows the model's algebraic implementation in GAMS. Section describes equilibrium conditions. Section 4 discusses issues about data aggregation and filtering.

Section 5 has a practical perspective with step-by-step instructions on how to install the GTAP7inGAMS package. I hope this provides a short learning curve for economists who wish to perform a few calculations using the GTAP dataset. This section describes ancillary GAMS programs which have been developed for use with the GTAP 7 dataset.

Finally, section 6 describes how the GTAP model is represented in GAMS. This material provides a short but complete overview of how the technology and preferences are calibrated along with GAMS code which performs this task.

2 The Model

The core GTAP model is a static, multi-regional model which tracks the production and distribution of goods in the global economy. In GTAP the world is divided into regions (typically representing individual countries), and each region's final demand structure is composed of public and private expenditure across goods. The model is based on optimizing behavior. Consumers maximize welfare subject to budget constraint with fixed levels of investment and public output. Producers combine intermediate inputs, and primary factors (skilled and unskilled labor, land, resources and physical capital) at least cost subject for given technology. The dataset includes a full set of bilateral trade flows with associated transport costs, export taxes and tariffs.

2.1 Benchmark Data and Accounting Identities

The economic structure underlying the GTAP dataset and model is illustrated in Figure 1. Symbols in this flow chart correspond to variables in the economic model. Y_{ir} portrays the production of good i in region r, C_r, I_r and G_r portray private consumption, investment and public demand, respectively. M_{jr} portrays the import of good j into region r. HH_r and $GOVT_r$ stand for representative household and government consumers, and FT_{sr} is the activity through which "sluggish" factors of production are allocated to individual sectors.

In this figure commodity and factor market flows appear as solid lines. Domestic and imported goods markets are represented by horizontal lines at the top of the figure. Domestic production (vom_{ir}) is distributed to exports ($vxmd_{irs}$), international transportation services (vst_{ir}), intermediate demand ($vdfm_{ijr}$), household consumption ($vdpm_{ir}$), investment

($vdipm_{ir}$)[3] and government consumption ($vdgm_{ir}$). The accounting identity in the gtap7 dataset is thus:

$$vom_{ir} = \sum_s vxmd_{irs} + vst_{ir} + \sum_j vdfm_{ijr} + vdpm_{ir} + vdgm_{ir} + vdim_{ir}$$

Imported goods which have an aggregate value of vim_{ir} enter intermediate demand ($vifm_{jir}$), private consumption ($vipm_{ir}$) and public consumption ($vigm_{ir}$). The accounting identity for these flows is thus:

$$vim_{ir} = \sum_j vifm_{ijr} + vipm_{ir} + vigm_{ir}$$

Inputs to Y_{ir} include intermediate inputs (domestic and imported), mobile factors of production ($vfm_{fir}, f \in m$), and sluggish factors of production ($vfr_{fir}, f \in s$). Factor earnings accrue to households. Factor market equilibrium is given by an identity relating the value of factor payments to factor income:

$$\sum_i vfm_{fir} = evom_{fr}$$

International market clearance conditions require that region r exports of good i (vxm_{ir} at the top of the figure) equal the imports of the same good in all trading partners ($vxmd_{irs}$ at the bottom of the figure):

$$vxm_{ir} = \sum_s vxmd_{irs}$$

Likewise, market clearance conditions apply for international transportation services. The supply demand balance in the market for transportation service j requires that the sum across all regions of service exports (vst_{ir}, at the top of the figure) equals the sum across all bilateral trade flows of service inputs ($vtwr_{jisr}$ at the bottom of the figure):

$$\sum_r vst_{jr} = \sum_{isr} vtwr_{jisr}$$

Tax revenues and transfers in Figure 1 are indicated by dotted lines. The flows labelled with $\mathcal{R}$ correspond to tax revenues.[4] Tax flows consist of indirect taxes on production/exports, ($\mathcal{R}^Y_{ir}$), consumption ($\mathcal{R}^C_r$), public demand ($\mathcal{R}^G_r$) and imports ($\mathcal{R}^M_{ir}$).

[3] In GTAP7inGAMS, $vdpm_{ir}$, $vdgm_{ir}$, and $vdipm_{ir}$ are implemented as subsets of $vdfm$ through the usage of the `"g"` index - see table 1

[4] These revenues do not appear as explicit variables in the GTAP database and are defined on the basis of expenditures and tax rates, as is subsequently described.

The regional budget constraint relates factor income, tax payments and net transfers from abroad (vb_r) to public and private consumption expenditure and investment:

$$vpm_r + vim_r + vgm_r = \sum_f evom_{fr} + \sum_i \mathcal{R}^Y_{ir} + \mathcal{R}^C_r + \mathcal{R}^G_r + \sum_i \mathcal{R}^M_{ir} + vb_r$$

To this point we have outlined two types of consistency conditions which are part of the GTAP database: market clearance (supply = demand for all goods and factors), and income balance (net income = net expenditure). A third set of identities involve net operating profits by all sectors in the economy. In the core GTAP model "production" takes place under conditions of perfect competition with constant returns to scale, hence there are no excess profits, and the cost of inputs must equal the value of outputs. This condition applies for each of the production sectors:

Y_{ir}: $\sum_f vfm_{fir} + \sum_j (vifm_{jir} + vdfm_{jir}) + \mathcal{R}^Y_{ir} = vom_{ir}$

M_{ir}: $\sum_s \left(vxmd_{isr} + \sum_j vtwr_{jisr}\right) + \mathcal{R}^M_{ir} = vim_{ir}$

C_r: $\sum_i (vdpm_{ir} + vipm_{ir}) + \mathcal{R}^C_{ir} = vpm_r$

G_r: $\sum_i (vdgm_{ir} + vigm_{ir}) + \mathcal{R}^G_{ir} = vgm_r$

I_r: $\sum_i vdim_{ir} = vim_r$

FT_{fr}: $evom_{fr} = \sum_i vfm_{fir} \quad f \in s$

Figure 1: Regional Economic Structure

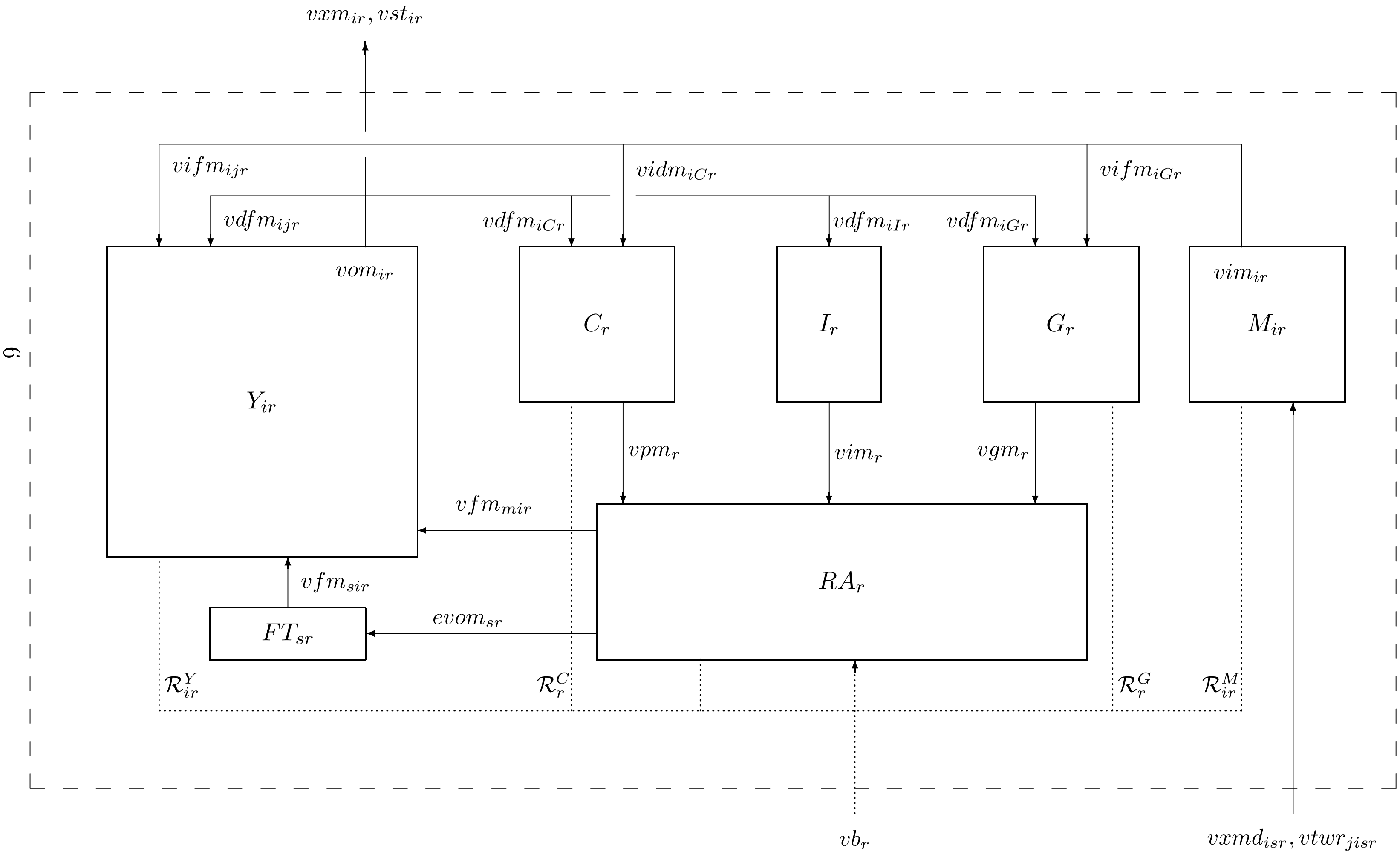


Table 1: Set Indices

i, j	Sectors, an aggregation of the 55 sectors in the GTAP 5 database
g	Production sectors i, plus private consumption `"c"`, public demand `"g"` and investment `"i"`
r	Regions, an aggregation of the 113 regions in the GTAP 7 database
f	Factors of production (consisting of *mobile factors*, $f \in m$, skilled labor, unskilled labor and capital, and *sluggish factors*, $f \in s$, agricultural land and other resources)

2.2 The Primal Formulation

The notation used in the model is summarized in the Tables 1 - 3. Table 1 defines the various dimensions which characterize an instance of the model, including the set of sectors/commodities, the set of regions, the set of factors of production. Set g is combines the production sectors i and private and public consumption demand (indices `"c"` and `"g"`) and investment demand (index `"i"`). It allows for a much tighter formulation of the model as they can all be conceived of "goods" produced in similar fashion. For exposition purposes however, the theoretical exposition of the model in the remainder of this paper will describe private consumption, public consumption and investment demand as stand alone components. Tables 2 and 3 display the concordance between the variables and their GAMS equivalents.

The GTAP database includes a 113 regions and 57 commodities, but dimensionality typically limits the number of regions and goods which can be included in a single model.[5] Table 2 defines the primal variables (activity levels) which define an equilibrium. The model determines values of all the variables except international capital flows, a parameter which would be determined endogenously in an intertemporal model.

Table 3 defines the relative price variables for goods and factors in the model. As is the case in any Shoven-Whalley model, the equilibrium conditions determine *relative* rather than *nominal* prices. One market equilibrium condition corresponds to each of the equilibrium prices.

The benchmark identities presented in the previous section indicate the market clearance, zero profit and income balance conditions which define the GTAP model. The displayed equations do not, however, characterize the behaviour of agents in the model. In the competitive equilibrium setting, the standard assumption of optimizing atomistic agents applies for both producers and consumers. Profit maximization in the constant returns to scale setting is equivalent to cost minimization subject to technical constraints. For sector Y_{ir} we characterize input choices as though they arose from minimization of

[5] Datasets are easily aggregated which permits empirical applications to be formulated and debugging with small dimensional models.

Table 2: Activity Levels

Var	Description	GAMS Variable	Bmk value
Y_{ir}	Production	`Y(i,r)`	`vom(i,r)`
C_r	Aggregate consumption D	`Y("c",r)`	`vom("c",r)`
G_r	Aggregate public D	`Y("g",r)`	`vom("g",r)`
I_r	Aggregate investment D	`Y("i",r)`	`vom("i",r)`
M_{ir}	Aggregate imports	`M("i",r)`	`vim(i,r)`
FT_{fr}	Factor transformation	`FT(f,r)`	
YT_j	Intl. transp. services	`YT(j)`	`vtw(j)`

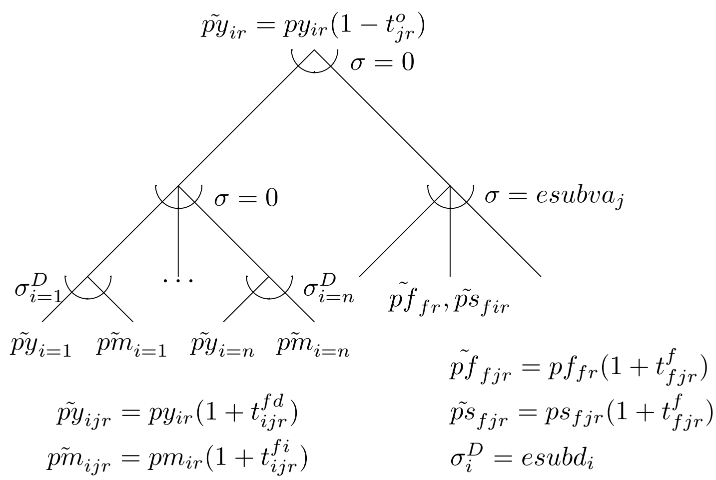
Table 3: Prices

p_r^C	Consumer price index
p_r^G	Public provision price index
p_{ir}^Y	Supply price, gross of indirect producer taxes
p_{ir}^M	Import price, gross of export taxes and tariffs.
p_j^T	Marginal cost of transport services
p_{fr}^F	Factor prices for labor, land and resources
p_{fir}^S	Price of sector-specific primary factors

Table 4: Tax and Subsidy Rates (net basis unless noted)

Tax		Symbol	GAMS Parameter
Output taxes (gross basis)		t^{o}_{ir}	`rto(i,r)`
Factor taxes		t^{f}_{fjr}	`rtf(f,j,r)`
Intermediate input taxes	Domestic	t^{fd}_{ijr}	`rtfd(i,j,r)`
	Imported	t^{fi}_{ijr}	`rtfi(i,j,r)`
Consumption taxes	Domestic	t^{pd}_{ir}	`rtfd("c",r)`
	Imported	t^{pi}_{ir}	`rtfi("c",r)`
Public demand taxes	Domestic	t^{gd}_{ir}	`rtfd("g",r)`
	Imported	t^{gi}_{ir}	`rtfi("g",r)`
Investment demand taxes	Domestic	t^{gd}_{ir}	`rtfd("i",r)`
	Imported	t^{gi}_{ir}	`rtfi("i",r)`
Export subsidies		t^{xs}_{isr}	`rtxs(i,s,r)`
Import tariffs		t^{ms}_{isr}	`rtms(i,s,r)`

Figure 2: Production Function : $Y_{ir} = F_{ir}(ddfm, difm, dfm)$



unit costs:[6]

$$\min_{difm,ddfm,dfm} \quad c^D_{ir} + c^M_{ir} + c^F_{ir} \tag{1}$$

s.t.

$$c^D_{ir} = \sum_j py_{jr}(1 + t^{fd}_{jir})ddfm_{jir}$$

$$c^M_{ir} = \sum_j pm_{jr}(1 + t^{fi}_{jir})difm_{jir}$$

$$c^F_{ir} = \sum_f (pf_{fr}|_{f\in m} + ps_{fir}|_{f\in s})(1 + t^f_{fir})dfm_{fir}$$

$$F_{ir}(ddfm, difm, dfm) = Y_{ir}$$

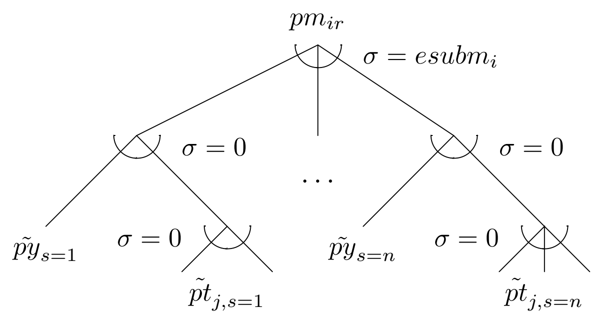
The production function appearing in these constraints is described by a nested constant-elasticity-of-substitution (CES) form, as shown in Figure 2. The specific source of tax revenue is indicated in this figure, consisting of output taxes, taxes on intermediate inputs and taxes on factor demands, all of which are applied on a ad-valorem basis.

The choice among imports from different trading partners is based on Armington's idea of regionally differentiated products. The following cost minimization problem:

$$\min_{dxmd,dtwr} \quad \sum_s (1 + t^{ms}_{isr}) \left(py_{is}(1 - t^{xs}_{isr})dxmd_{isr} + \sum_j pt_j dtwr_{jisr} \right) \tag{2}$$

[6] Decision variables appearing in the primal model correspond to the benchmark data structures with the initial "v" replaced by "d". Hence, while $vdfm_{jir}$ represents benchmark intermediate demand for good j in the production of good i in region r, $ddfm_{jir}$ represents the corresponding decision variable in the equilibrium model.

Figure 3: Armington Aggregation : $M_{ir} = A_{ir}(dxmd, dtwr)$



$$\tilde{py}_{is} = py_{is}(1 - t^{xs}_{isr})(1 + t^{ms}_{isr})$$
$$\tilde{pt}_{js} = pt_j(1 + t^{ms}_{isr})$$

s.t.

$$A_{ir}(dxmd, dtwr) = M_{ir}$$

The import aggregation function portrayed by A in (2) is described by the nested CES-Leontief function shown in Figure 3. Transportation services enter on a proportional basis with imports from different countries, reflecting differences in unit transportation margins across different goods and trading partners. Substitution at the top level in an Armington composite involves trading off of both imported goods and associated transportation services. Trade flows are subject to export subsidies and import tariffs, with subsidies paid by government in the exporting region, and tariffs collected by government in the importing region.

Private consumption consistent with utility maximization is portrayed by minimization of the cost of a given level of aggregate consumption[7]:

$$\min_{ddpm, dipm} \quad \sum_i py_{ir}(1 + t^{pd}_{ir})ddpm_{ir} + pm_{ir}(1 + t^{pi}_{ir})dipm_{ir} \tag{3}$$

s.t.

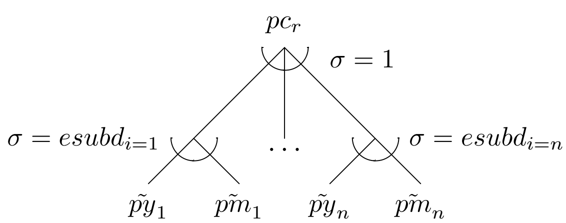
$$H_r(ddpm, dipm) = C_{ir}$$

Final demand in the core model is characterized by a Cobb-Douglas tradeoff across composite goods which include both domestic and imported inputs. The nested CD-CES function is displayed in Figure 4.

Land and natural resources are portrayed as sector-specific factors of production supplied through constant-elasticity-of-transformation (CET) production function allocates composite factors to sectoral markets. The supply of sectoral factors of production are

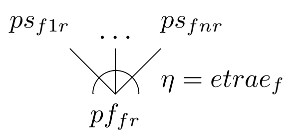
[7] In the model code, as private consumption is integrated within the `"g"` set with all other goods (index `"c"`), $vdpm_{ir}$ and $vipm_{ir}$ correspond to `vdfm(i,"c",r)` and `vifm(i,"c",r)`, respectively

Figure 4: Private Consumption : $C_r = H_r(ddpm, dipm)$



$$\tilde{py}_{ir} = py_{ir}(1 + t_{ir}^{pd})$$
$$\tilde{pm}_{ir} = pm_{ir}(1 + t_{ir}^{pi})$$

Figure 5: Sluggish Factor Transformation : $FT_f = \Gamma_f(dfm)$



portrayed as arising from the following profit-maximization problem:

$$\max_{dfm} \quad \sum_j dfm_{sjr} ps_{sjr} \tag{4}$$

s.t.

$$\Gamma_{sr}(dfm) = evom_{sr}$$

This CET function (Γ) is illustrated in Figure 5.

International transportation services are provided as a aggregation of transportation services exported from countries throughout the world. The aggregation of transportation services is represented in the model by a cost minimization problem:

$$\min_{dst} \quad \sum_r py_{ir} dst_{ir}$$

s.t.

$$T_i(dst) = YT_i$$

The aggregation function which combines transport service exports from multiple regions. The functional form which aggregates services from different regions is illustrated in Figure 6.

Public consumption in the model is represented as a fixed coefficient (Leontief) aggregation of domestic-import composites. This formulation introduces substitution at the second level between domestic and imported inputs while holding sectal commodity aggregates constant. Figure 7 illustrates the functional form [8].

[8] Again, as public consumption is integrated within the `"g"` set with all other goods, $vdgm_{ir}$ and $vigm_{ir}$ correspond to `vdfm(i,"g",r)` and `vifm(i,"g",r)`, respectively

Figure 6: International Transportation Services : $YT_j = T_j(dst)$

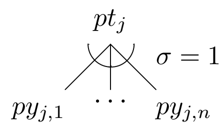
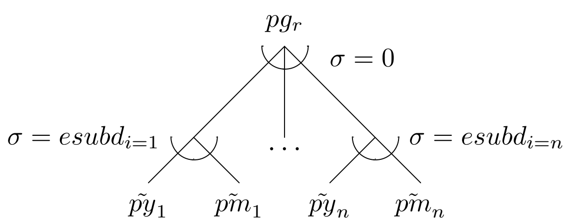


Figure 7: Public Consumption : $G_r = G_r(ddgm, digm)$



$$\tilde{py}_{ir} = py_{ir}(1 + t_{ir}^{gd})$$
$$\tilde{pm}_{ir} = pm_{ir}(1 + t_{ir}^{gi})$$

3 Equilibrium Conditions

Equilibrium conditions in this version of the model are presented in a somewhat different format than in Rutherford [1997]. The formulation presented here is based on an explicit "dual" approach. (See Dixit and Norman [1992] or Varian [1992] for introductions to the dual approach.)

An Arrow-Debreu model concerns the interaction of consumers and producers in markets. Mathiesen [1985] proposed a representation of this class of models in which two types of equations define an equilibrium: zero profit and market clearance. The corresponding variables defining an equilibrium are activity levels (for constant-returns-to-scale firms) and commodity prices.[9] Here we extend Mathiesen's framework with a third class of variables corresponding to consumer income levels. Commodity markets encompass primary endowments of households with producer outputs. In equilibrium the aggregate supply of each good must be at least as great as total intermediate and final demand. Initial endowments are exogenous. Producer supplies and demands are defined by producer activity levels and relative prices. Final demands are determined by market prices.

Economists who have worked with conventional textbook equilibrium models can find Mathiesen's framework to be somewhat opaque because many quantity variables are not explicitly specified in the model. Variables such as final demand by consumers, factor

[9]Under a maintained assumption of perfect competition, Mathiesen may characterize technology as CRTS without loss of generality. Decreasing returns are accommodated through introduction of a specific factor, while increasing returns are inconsistent with the assumption of perfect competition. In this environment zero excess profit is consistent with free entry for atomistic firms producing an identical product.

demands by producers and commodity supplies by producers, are defined implicitly in Mathiesen's model. For example, given equilibrium prices for primary factors, consumer incomes can be computed, and given income and goods prices, consumers' demands can then be determined. The consumer demand functions are written down in order to define an equilibrium, but quantities demanded need not appear in the model as separate variables. The same is true of inputs or outputs from the production process: relative prices determine conditional demand, and conditional demand times the activity level represents market demand. Omitting decisions variables and suppressing definitional equations corresponding to intermediate and final demand provides significant computational advantages at the cost of a somewhat more complex model statement. [10]

3.1 Zero profit (arbitrage) conditions

All production activites in the model are represented by constant-returns-to-scale technologies, and markets are assumed to operate competitively with free entry and exit. As a conseqence, equilibrium profits are driven to zero and the price of output reflects the cost of inputs. The following sets of equations relating output price to marginal cost are part of the definition of an equilibrium.[11]

The calculation of unit cost and unit revenue functions involves the definition of a number of ancillary variables. I follow the notational convention that ancillary variables, those which do not appear explicitly as GAMS variables, are defined in un-numbered equations. This indicates that they are "optional" in the sense that they may be substituted out of the central system of equations which define the market clearance, zero profit and income balance equations. Price indices, unit cost and unit revenue indices, tax revenue equations, and compensated demand and supply functions are "invisible" in the MPSGE implementation of the model.

N.B. We use the symbol θ to portray value shares from the base year data. In many cases subscripts on these value shares are omitted in order to economize on notation.

3.1.1 Sectoral production (`Y(g,r)`)

The unit cost of value-added is a a CES composite of skilled and unskilled labor, land, resources and capital inputs to production, gross of taxes. Factor inputs may be sector-

[10]The alegraic implementation of the GTAP model are provided here in GAMS/MCP and GAMS/CNS. (In GAMS, MCP refers to the "mixed complementarity format" and CNS is a "constrained nonlinear system".) These formats can be used interchangably for GTAP models, hence the PATH or CONOPT solvers would both be suitable solvers. MILES could also be used, but it may be incapable of processing large scale instances. I provide two alternative versions of the algebraic models. The smaller-dimensional dense implementation uses GAMS-F, a function preprocessor for GAMS which avoids the introduction of intermediate variables representing unit cost and demand functions. The larger-dimensional sparse implementation includes the intermediate variables and associated equations.

[11]This model is essentially a nonlinear system of equations which corresponds to a special form of complementarity. We retain consistency with the complementarity format by expressing arbitrage conditions as "oriented equations" with marginal cost on the LHS and marginal revenue on the RHS.

specific (sluggish) or flexible:

$$p^{pf}_{fjr} = \begin{cases} p^F_{fr}\frac{(1+t^f_{fjr})}{1+\bar{t}^f_{fjr}} & f \in m \\ p^S_{fjr}\frac{(1+t^f_{fjr})}{1+\bar{t}^f_{fjr}} & f \in s \end{cases}$$

and the unit cost function is given by:

$$c^f_{jr} = \left(\sum_f \theta_f \left(p^{pf}_{fjr}\right)^{1-\sigma}\right)^{1/(1-\sigma)}$$

The user cost of intermediate inputs inputs, differs from the market price, due to the presence of taxes on intermediate inputs.

$$p^d_{ijr} = p^Y_{ir}\frac{1+t^{fd}_{ijr}}{1+\bar{t}^{fd}_{ijr}}$$

$$p^i_{ijr} = p^M_{ir}\frac{1+t^{fi}_{ijr}}{1+\bar{t}^{fi}_{ijr}}$$

A CES cost function then describes the minimum cost of a bundle of domestic and imported inputs to production, based on benchmark value shares and an elasticity of substition $\sigma = esubd_i$:

$$c^i_{ijr} = \left(\theta_d(p^d_{ijr})^{1-\sigma} + (1-\theta_d)(p^i_{ijr})^{1-\sigma}\right)^{1/(1-\sigma)}$$

Unit cost is then a Leontief (linear) composite of the costs of intermediate and primary factor inputs, based on base year value shares:

$$c^y_{jr} = \sum_i \theta_i c^i_{ijr} + \theta_f c^f_{jr}$$

Having formulated the unit cost function, it is possible to compactly portray the zero profit condition for y_{jr}. In equilibrium, the marginal cost of supply equals the market price, net of taxes:

$$cy_{jr} = py_{jr}\frac{1-t^o_{jr}}{1-\bar{t}^o_{jr}} \tag{5}$$

Compensated demand functions related to y_{ir} include those which define domestic and imported intermediates.[12]

$$ddfm_{ijr} = y_{jr}\text{vdfm}_{ijr}\left(\frac{ci_{ijr}}{p^d_{ijr}}\right)^\sigma$$

and

$$difm_{ijr} = y_{jr}\text{vifm}_{ijr}\left(\frac{ci_{ijr}}{p^i_{ijr}}\right)^\sigma$$

[12]These demand functions are needed in formulating market clearance conditions below.

We also will subsequently need to define demands for primary factors in sector y_{ir}:

$$dfm_{fjr} = y_{jr} vfm_{fjr} \left(\frac{cf_{jr}}{p^{pf}_{fjr}} \right)^{\sigma}$$

3.1.2 International transportation services (`YT(j)`)

The unit cost of a transportation service depends on the benchmark value shares of region-specific services through a Cobb-Douglas cost function. Under perfect competition with free entry, the unit cost of international transport services equals the equilibrium market price:

$$\prod_r (py_{jr})^{\theta_j} = pt_j \tag{6}$$

The export demand for transportation service j from region r may then be written as a closed-form function of relative prices and the aggregate provision of those services (yt_j):

$$dst_{jr} = yt_j \mathrm{vst}_{jr} \frac{pt_j}{py_{jr}}$$

3.1.3 Private demand (`Y("c",r)`)

In the core model the consumer price index represents a Cobb-Douglas composite of the domestic and imported goods prices gross of tax. The price indices for domestic and imported goods are given by:

$$p^{dc}_{ir} = p^{y}_{ir} \frac{1 + t^{pd}_{ir}}{1 + \bar{t}^{pd}_{ir}}$$

and

$$p^{ic}_{ir} = p^{m}_{ir} \frac{1 + t^{pi}_{ir}}{1 + \bar{t}^{pi}_{ir}}$$

The unit cost of good i is a CES cost function defined over these price indices, based on benchmark value shares and an elasticity of substitution $\sigma = esubd(i)$:

$$p^{c}_{ir} = \left(\theta (p^{dc}_{ir})^{1-\sigma} + (1-\theta)(p^{ic}_{ir})^{1-\sigma} \right)^{1/(1-\sigma)}$$

The consumer price of aggregate supply is defined by a Cobb-Douglas price index defined over price indices of domestic and imported varieties:

$$\prod_i (p^{c}_{ir})^{\theta_i} = pc_r \tag{7}$$

Consumer demands for domestic and imported goods can be expressed on the basis of the aggregate consumption level and the prices of domestic and imported goods, gross of tax:

$$ddpm_{ir} = c_r \mathrm{vdpm}_{ir} \left(\frac{p^{c}_{ir}}{p^{dc}_{ir}} \right)^{\sigma} \frac{pc_r}{p^{c}_{ir}}, and$$

$$dipm_{ir} = c_r \mathrm{vipm}_{ir} \left(\frac{p^{c}_{ir}}{p^{ic}_{ir}} \right)^{\sigma} \frac{pc_r}{p^{c}_{ir}}.$$

3.1.4 Government demand (`Y("g",r)`)

Public expenditure in the core model is a fixed-coefficient aggregate of Armington composite goods. Within each composite domestic and imported goods trade off with a constant elasticity of substitution. The unit price indices for domestic and imported goods are given by:

$$p_{ir}^{dg} = p_{ir}^{y} \frac{1 + t_{ir}^{dg}}{1 + \bar{t}_{ir}^{gd}}$$

and

$$p_{ir}^{ig} = p_{ir}^{m} \frac{1 + t_{ir}^{ig}}{1 + \bar{t}_{ir}^{ig}}$$

The composite price of the ith good is then:

$$p_{ir}^{g} = \left(\theta (p_{ir}^{dg})^{1-\sigma} + (1-\theta)(p_{ir}^{ig})^{1-\sigma}\right)^{1/(1-\sigma)}$$

The cost of public services (G_r) is defined by the Leontief cost coefficients:

$$\sum_i \theta_i p_{ir}^{g} = pg_r \tag{8}$$

Government demands for domestic and imported goods is written as:

$$ddgm_{ir} = g_r \text{vdgm}_{ir} \left(\frac{p_{ir}^{g}}{p_{ir}^{dg}}\right)^{\sigma} \frac{pg_r}{p_{ir}^{g}}$$

and

$$digm_{ir} = g_r \text{vigm}_{ir} \left(\frac{p_{ir}^{g}}{p_{ir}^{ig}}\right)^{\sigma} \frac{pg_r}{p_{ir}^{g}}$$

3.1.5 Aggregate imports (`M(i,r)`)

Import cost index which applies export taxes, trade and transport margins and import tariffs to the producer supply prices in exporting regions:

$$py_{isr}^{m} = p_{is}^{y} \frac{(1 - t_{isr}^{xs})(1 + t_{isr}^{ms})}{(1 - \bar{t}_{isr}^{xs})(1 + \bar{t}_{isr}^{ms})}$$

and the unit price of transporation services is given by:

$$pt_{jisr}^{m} = p_{j}^{T} \frac{1 + t_{isr}^{ms}}{1 + \bar{t}_{isr}^{ms}}$$

Transportation margins enter as fixed coefficients with bilateral trade flows, so the unit delivered price is a convex combination of the unit prices with weights corresponding to base year value shares:

$$pyt_{isr}^{m} = \theta py_{isr}^{m} + \sum_j \theta_j^{T} pt_{jisr}^{m}$$

Having formed a price index for bilateral imports from region s, the CES cost index can be defined on the basis of value shares and the elasticity of substitution across imports from different regions, $\sigma = esubm(i)$:

$$cim_{ir} = \left(\sum_s \theta_s (pyt^m_{isr})^{1-\sigma)} \right)^{1/(1-\sigma)}$$

The import activity (m_{ir}) has a zero profit condition which relates the unit cost of imports to the market price of the import aggregate:

$$cim_{ir} = pm_{ir} \tag{9}$$

Import demands can then be expressed in closed form on the basis of the sectoral import level and the tax-inclusive prices of imports and transportation services:

$$dxmd_{isr} = m_{ir} \mathrm{vxmd}_{isr} \left(\frac{pm_{ir}}{pyt^m_{isr}} \right)^{\sigma}$$

and

$$dtwr_{jisr} = m_{ir} \mathrm{vtwr}_{jisr} \left(\frac{pm_{ir}}{pyt^m_{isr}} \right)^{\sigma}$$

3.1.6 Sluggish Factor Transformation (`FT(f,r)`)

The unit value of sector-specific factors is defined as a CET revenue function based on the base year value shares (θ_j)

$$pvfm_{fr} = \left(\sum_j \theta_j ps^{1+\eta}_{fjr} \right)^{1/(1+\eta)} \qquad f \in s$$

This defines the profit-maximizing allocation of factors to individual sectors. In equilibrium, the unit value of the aggregate factor is equal to the maximum unit earnings:

$$pf_{fr} = pvfm_{fjr} \quad f \in s \tag{10}$$

3.2 Market clearance

Supply-demand conditions apply to all goods and factors. Benchmark demand and supply quantities appear as scale factors in many of these equations, typically multiplied by activity levels which are equal to unity in the reference equilibrium.[13]

[13] While not crucial for representation of the model as a nonlinear system of equations, I follow the MCP convention in writing out the market clearance conditions. The equations are "oriented", with supply variables on the LHS and demands on the RHS. Hence, the sense of the equation is *supply* $\geq$ *demand*. In the core model equilibrium prices should always be positive, but in extensions of the standard model it might be quite common to introduce inequalities and complementary slackness, in which case the proper orientation of the equations is essential. Hence, in equilibrium should the price of a good be zero, economic equilibrium is then consistent with an associated market in which *supply* $>$ *demand*.

3.2.1 Private and public consumption (`P("c",r)` and `P("g",r)`)

Consumer demand in region r in the reference equilibrium is `vpm(r)` and public demand is `vgm(r)` [14]:

$$c_r \mathrm{vpm}_r + g_r \mathrm{vgm}_r = RA_r \tag{11}$$

3.2.2 Firm output (`P(i,r)`)

Aggregate output of good i in region r in the reference equilibrium is `vom(i,r)`:

$$y_{ir} \mathrm{vom}_{ir} = \sum_j ddfm_{ijr} + ddpm_{ir} + dipm_{ir} + ddgm_{ir} + \sum_s dxmd_{irs} + dst_{ir} \tag{12}$$

3.2.3 Composite imports (`PM(i,r)`)

The aggregate value of imports of good i in region r in the reference equilibrium is `vim(i,r)`:

$$m_{ir} \mathrm{vim}_{ir} = \sum_j difm_{ijr} + dipm_{ir} + digm_{ir} \tag{13}$$

3.2.4 Transport services (`PT(j)`)

The aggregate demand (and supply) for transport service j in the benchmark equilibrium is `vtw(j)`:

$$yt_j \mathrm{vtw}_j = \sum_{isr} dtwr_{jisr} \tag{14}$$

3.2.5 Primary factors (`PF(f,r)`)

The aggregate demand (and supply) of primary factor f in region r is `evom(f,r)`:

$$\mathrm{evom}_{fr} = \begin{cases} \sum_j dfm_{fjr} & f \in m \\ \mathrm{evom}_{fr} ft_{fr} & f \in s \end{cases} \tag{15}$$

3.2.6 Specific factors (`PS(f,j,r)`)

The net value of benchmark payments to factor f in sector j in region r is `vfm(f,j,r)`:

$$\mathrm{vfm}_{fjr} \left(\frac{ps_{fjr}}{pf_{fr}} \right)^{\eta} = dfm_{fjr} \tag{16}$$

[14] `vom("c",r)` and `vom("g",r)` in GAMS code

3.3 Regional budget (RA(r))

Private and public incomes are given by :

$$RA_r = \sum_f pf_{fr}\mathrm{evom}_{fr} + pc_n\mathrm{vb}_r - \sum_i py_{ir}\mathrm{vdim}_{ir} + \mathcal{R}_r \tag{17}$$

The base year current account deficit in region r is `vb(r)`, and region $r = n$ corresponds to the "numeraire region" who's consumption prices denominates international capital flows.

Tax revenue in region r consists of output taxes, intermediate demand taxes, factor taxes, final demand taxes, import tariffs and export subsidies:

$$\mathcal{R}_r = \mathcal{R}_r^o + \mathcal{R}_r^{fd} + \mathcal{R}_r^{fi} + \mathcal{R}_r^f + \mathcal{R}_r^{pd} + \mathcal{R}_r^{pi} + \mathcal{R}_r^{gd} + \mathcal{R}_r^{gi} - \mathcal{R}_r^{xs} + \mathcal{R}_r^{ms} \tag{18}$$

Each of these componets of tax revenue can be calculated as an ad-valorem or proportional tax rate times a market price times the quantity demanded or produced.

Taxes related to y_{ir} include output taxes:

$$\mathcal{R}_r^o = \sum_j t_{jr}^o \mathrm{vom}_{jr} py_{jr} y_{jr},$$

tax revenue from intermediate inputs:

$$\mathcal{R}_r^{fd} = \sum_{ij} t_{ijr}^{fd} py_{ir} ddfm_{ijr}, and$$

$$\mathcal{R}_r^{fi} = \sum_{ij} t_{ijr}^{fi} pm_{ir} difm_{ijr},$$

and factor tax revenue:

$$\mathcal{R}_r^f = \sum_{fj} t_{fjr}^f pf_{fr} dfm_{fjr}.$$

Taxes on household consumption of domestic and imported goods are:

$$\mathcal{R}_r^{pd} = \sum_i t_{ir}^{pd} py_{ir} ddpm_{ir},$$

and

$$\mathcal{R}_r^{pi} = \sum_i t_{ir}^{pi} pm_{ir} dipm_{ir}.$$

Taxes on public demand for domestic and imported goods are:

$$\mathcal{R}_r^{gd} = \sum_i t_{ir}^{gd} py_{ir} ddgm_{ir}$$

and

$$\mathcal{R}_r^{gi} = \sum_i t_{ir}^{gi} pm_{ir} digm_{ir}.$$

Export subsidies (paid by the government in the exporting region) are:

$$\mathcal{R}_r^{xs} = \sum_{is} t_{irs}^{xs} py_{ir} dxmd_{irs}$$

and import tariff revenues are given by:

$$\mathcal{R}_r^{ms} = \sum_{is} t_{isr}^{ms} \left(py_{is}(1 - t_{isr}^{xs}) dxmd_{isr} + \sum_j pt_j dtwr_{jisr} \right)$$

4 Data

4.1 Datasets

GTAPinGAMS datasets are stored in the GAMS Data eXchange (GDX) format. Data stored in this format may be freely transferred to header array format using Mark Horridge's `gdx2har.exe` program. Data may also be transferred to Excel using any of several free utilities provided by GAMS Development Corporation (see, e.g., `gdxrw.exe`). Any GTAPinGAMS dataset may be aggregated into fewer regions, sectors and primary factors. This permits a modeller to do preliminary model development using a small dataset to ensure rapid response and a short debug cycle. After having implemented a small model, it is then a simple matter to expand the number of sectors and/or regions in order to obtain a more precise empirical estimate.

All GTAP datasets are defined in terms of three primary sets: i, the set of sectors and produced commodities, r the set of countries and regions, and f the set of primary factors. Table 6.6 presents the identifiers for the 59 GTAP 7 sectors in their most disaggregate form. These sectors may be aggregated freely to produce more compact dataset.

Regional identifiers in the full dataset correspond to standard UN three-character country codes. Users can define their own aggregations of the GTAP data and use any labels to describe regions. For technical reasons, if a GTAP dataset is to be used with MPSGE, then regional identifiers can have at most 4 characters. Table 7 presents the three-character identifiers which I normally use for primary factors.

4.2 Data filtering

The GTAP source data presents substantial challenges for calibrated models processed using direct solution methods (e.g., PATH, CONOPT or MINOS). In my experience, most numerical problems with GTAPinGAMS models can be traced to the presence of large numbers of small coefficients in the source data. These coefficients portray economic flows which are a negligible share of overall economic activity, yet impose a significant computational burden during matrix factorization.

GTAP7inGAMS includes a GAMS program (FILTER.GMS) which removes small values and re-calibrates the resulting dataset. An input to this program (TOL) determines the filter tolerance. Values of TOL would normally range from 0.0001 to 0.01. Smaller values of TOL retain a larger number of small coefficients in the filtered dataset. Filtering makes a GTAP database smaller, as illustrated in figure 4.2, in which it can be seen that filtering reduces the size of a GTAP database by somewhere between 20% and 80%, depending on the filtering tolerance.

The filtering procedure has differential impacts on various components of the database. The largest proportional reduction in parameter density occurs in public demand, arrays

Table 5: Data Filtering Tolerance Results

	0001	**001**	**005**	**01**
Δ % Total	-24%	-37%	-53%	-62%
% Total				
Δ vxmd	-4%	-8%	-13%	-16%
Δ vtwr	-0%	-2%	-6%	-9%
Δ vdfm	-6%	-9%	-11%	-11%
Δ vifm	-8%	-10%	-12%	-12%
Δ rtxs	-2%	-3%	-4%	-5%
% Δ vxmd	-17%	-33%	-51%	-61%
% Δ vtwr	-1%	-9%	-27%	-40%
% Δ vdfm	-45%	-62%	-77%	-82%
% Δ vifm	-57%	-75%	-86%	-89%
% Δ rtxs	-19%	-36%	-53%	-63%
% Δ rtms	-8%	-19%	-37%	-49%
% Δ evt	-56%	-61%	-66%	-69%
% Δ enco2	-53%	-55%	-58%	-61%
% Δ evd	-35%	-40%	-45%	-50%
% Δ eco2	-37%	-42%	-48%	-52%
% Δ vfm	-2%	-6%	-10%	-12%

vdgm and vigm. There are also substantial reductions in the density of international trade and intermediate input arrays.

Rounding to zero depends on relative tolerances. Domestic demands which are smaller than TOL/10 times total demand are set to zero. Bilateral trade flows are filtered on the basis of relative tolerances which account for the sizes of both importing and exporting economies. Most of the reduction in nonzeros results from the elimination of small intermediate inputs and bilateral trade flows, as can be seen in Table 4.2.

Depending on the nature of the policy question under investigation, different filtering targets may be adopted. Individual researchers may have their own opinions about how to select parameter values should be rounded to zero. The version of `FILTER.GMS` provided with the GTAP7inGAMS distribution is intended to provide a starting point for this step in the dataset development process.

5 Practicalities

5.1 System Requirements

You will need to have the following:

- A computer.
- A GAMS system, Version 23.1 or newer.
- The PATH or CONOPT solvers.[15]
- MPSGE subsystem (optional)

5.2 Getting Started

The GTAPinGAMS package is distributed as a zip file containing the directory structure and GAMS programs which unzips into a new directory.

GAMS source code and several free aggregate datasets are provided with the distribution directory. There GTAP source data is not distributed with the GTAPinGAMS system. In order to generate large scale models with the GTAPinGAMS tools, it is necessary to obtain the GTAP 7 distribution data files GSDDAT.HAR, GSDPAR.HAR, and GSDSET.HAR.

Here are the steps involved in installing GTAP7inGAMS:

1. Create an empty root directory for the gtap7 model.
2. Unzip GTAP7inGAMS.ZIP in the root directory.
3. Install the gtap7 data and run the build script (optional)
4. Test the aggregation routine
5. Test the recalibration routine
6. Check benchmark consistency and generate an echoprint

[15] You save a lot of time with FILTER.GMS if you have a copy of CPLEX, MOSEK or some other QCP solver.

5.3 Directory Structure

When you install GTAP7inGAMS, the root directory is empty, and all files reside in one of following six second-level subdirectories:

- BUILD

  Contains GAMS programs for dataset extraction FLEX2GDX.GMS, filtering FILTER.GMSand aggregation (GTAPAGGR.GMS). The MAKE.BAT batch file runs these files and creates a useable GTAPinGAMS data file from the original data.

- DEFINES

  Contains mapping files for aggregations of gtap7 datasets. Files ending with .MAP define an aggregation in terms of the source dataset and mappings from sets in the source to sets in the target.

- MODELS

  Contains template GAMS programs illustrating how the GTAP data can be GAMS programs. These include two model files:

  MRTMGE.GMS The standard model specified as an mixed complementarity model using an MPSGE representation of demand and supply functions.

  MRTMCP.GMS The standard model specified as an mixed complementarity model using GAMS with the GAMS-F function preprocessor. The model is specified in both MCP and CNS formats.

- INCLIB

  Contains batinclude routines accessed by GAMS programs in subdirectories BUILD and MODELS.

- GTAPDATA

  Contains source data files, files with a *.HAR extension. These are GTAP datasets in the original format. If you have the GTAP distribution file. The following files are accessed:

  GSDDAT.HAR Copied from the Flexagg7 directory.

  GSDPAR.HAR Copied from the Flexagg7 directory.

  GSDSET.HAR Copied from the Flexagg7 directory.

  GSDVOLE.HAR Copied from the Flexagg7 directory.

  GTAP_CO2_V7.HAR Obtained from GTAP Resource #1143.

  GTP_NCO2_MMTCEQ_V7.HAR Obtained from GTAP Resource #3191 (optional).

- DATA

  Contains constructed data files. Files with the *.GDX extension are GAMSinGTAP datasets for alternative aggregations. If you have the GTAP distribution file, all files in the DATA subdirectory may be regenerated.

5.4 Create an aggregated database file

These are the steps to follow in order to create a usable GDX format file from the original GTAP HAR files.

1. copy GSDPAR.HAR, GSDDAT.HAR and GSDTAX.HAR from the FLEXAGG package to the DATA directory
2. a mapping file containing a complete mapping of regions, sectors and factors must be included in the DEFINES directory
3. Run flex2gdx.gms - it outputs a GSD.GDX, a GDX version of the GTAP database
4. Run filter.gms by specifying the desired tolerance level through global variable TOL - it outputs GSD%TOL%.GDX, a filtered version of the GTAP database
5. Run gtapaggr.gms specifiying the outputed GDX file from the previous step as the SOURCE variable, and the name of the mapping file as the TARGET variable - it outputs %TARGET%.GDX an aggregated version of the GTAP database which is ready for use

6 The GAMS Code

6.1 Declarations

```
$title  GTAP7inGAMS Model in GAMS/MCP Algebraic Format

* !! this model requires GAMS version 23.1 at least in order to deal with macros correctly

$set ds test
$batinclude gtap7data

parameter       esub(g)         Top-level elasticity indemand /C 1/;

alias (j,jj), (g,gg), (f,ff);

nonnegative variables
        Y(g,r)          Supply
        M(i,r)          Imports
        YT(j)           Transportation services
        FT(f,r)         Specific factor transformation

        P(g,r)          Domestic output price
        PM(j,r)         Import price
        PT(j)           Transportation services
        PF(f,r)         Primary factors rent
        PS(f,g,r)       Sector-specific primary factors
        RA(r)           Representative agent;

equations
        prf_y(g,r)              Supply
        prf_m(i,r)              Imports
```

```
        prf_yt(j)               Transportation services
        prf_ft(f,r)             Factor transformation

        mkt_p(g,r)              Domestic output price
        mkt_pm(j,r)             Import price
        mkt_pt(j)               Transportation services
        mkt_pf(f,r)             Primary factors
        mkt_ps(f,j,r)           Specific factor

        inc_ra(r)               Representative agent;
```

6.2 Zero profit (arbitrage) conditions

6.2.1 Sectoral production – Y(g,r)

```
*       Define some macros which diagnose the functional form:

$macro  Leontief(sigma)         (yes$(round(sigma,2)=0))
$macro  CobbDouglas(sigma)      (yes$(round(sigma-1,2)=0))
$macro  CES(sigma)              (yes$(round(sigma-1,2)>0 and round(sigma,2)>0))


*       ------------------------------------------------------------------------------
*       MPSGE:

* $prod:Y(g,r)$vom(g,r) s:esub(g)    i.tl:esubd(i)  va:esubva(g)
*       o:P(g,r)        q:vom(g,r)      a:RA(r)  t:rto(g,r)
*       i:P(i,r)        q:vdfm(i,g,r)   p:(1+rtfd0(i,g,r)) i.tl:  a:RA(r) t:rtfd(i,g,r)
*       i:PM(i,r)       q:vifm(i,g,r)   p:(1+rtfi0(i,g,r)) i.tl:  a:RA(r) t:rtfi(i,g,r)
*       i:PS(sf,g,r)    q:vfm(sf,g,r)   p:(1+rtf0(sf,g,r))  va:    a:RA(r) t:rtf(sf,g,r)
*       i:PF(mf,r)      q:vfm(mf,g,r)   p:(1+rtf0(mf,g,r))  va:    a:RA(r) t:rtf(mf,g,r)


*       Benchmark value shares:

parameter       thetaf(f,g,r)   Factor share of value added,
                thetad(i,g,r)   Domestic share of intermediate input,
                thetai(i,g,r)   Import share of intermediate input,
                theta_f(g,r)    Value added share of sectoral output;

thetaf(f,g,r)$sum(ff,vfm(ff,g,r)*(1+rtf0(ff,g,r)))
        = vfm(f,g,r)*(1+rtf0(f,g,r)) / sum(ff,vfm(ff,g,r)*(1+rtf0(ff,g,r)));

thetad(i,g,r)$(vdfm(i,g,r)*(1+rtfd0(i,g,r)) + vifm(i,g,r)*(1+rtfi0(i,g,r)))
        = vdfm(i,g,r)*(1+rtfd0(i,g,r)) /
                (vdfm(i,g,r)*(1+rtfd0(i,g,r)) + vifm(i,g,r)*(1+rtfi0(i,g,r)));
thetai(i,g,r)$vom(g,r)
        = (vdfm(i,g,r)*(1+rtfd0(i,g,r)) + vifm(i,g,r)*(1+rtfi0(i,g,r))) / vom(g,r);

theta_f(g,r)$vom(g,r) = sum(ff,vfm(ff,g,r)*(1+rtf0(ff,g,r))) / vom(g,r);

display thetai, theta_f;
```

```
*       User cost indices for factors, domestic and imported
*       intermediate inputs:

$macro P_PF(f,g,r) (((PF(f,r)$mf(f)+PS(f,g,r)$sf(f))*(1+rtf(f,g,r)) \
                / (1+rtf0(f,g,r)))$thetaf(f,g,r)  + 1$(thetaf(f,g,r) = 0))
$macro P_D(i,g,r)  ((P(i,r)*(1+rtfd(i,g,r)) \
                / (1+rtfd0(i,g,r)))$thetad(i,g,r) + 1$(thetad(i,g,r)=0))
$macro P_I(i,g,r)  ((PM(i,r)*(1+rtfi(i,g,r)) \
                / (1+rtfi0(i,g,r)))$(1-thetad(i,g,r)) + 1$(thetad(i,g,r)=1))

*       Compensated cost functions:

$if defined f_ $abort "The CF(g,r) macro requires a uniquely defined alias for f."
alias (f,f_);
$macro CF(g,r) ( \
        (sum(f_, thetaf(f_,g,r)*P_PF(f_,g,r)))$Leontief(esubva(g)) + \
        (prod(f_, P_PF(f_,g,r)**thetaf(f_,g,r)))$CobbDouglas(esubva(g)) + \
        (sum(f_, thetaf(f_,g,r)*P_PF(f_,g,r)**(1-esubva(g)))**(1/(1-esubva(g))))$CES(esubva(g)))

$macro CI(i,g,r) ( \
        (thetad(i,g,r)*P_D(i,g,r) + (1-thetad(i,g,r))*P_I(i,g,r))$Leontief(esubd(i)) + \
        (P_D(i,g,r)**thetad(i,g,r) * P_I(i,g,r)**(1-thetad(i,g,r)))$CobbDouglas(esubd(i)) + \
        ((thetad(i,g,r) *P_D(i,g,r)**(1-esubd(i)) + \
        (1-thetad(i,g,r))*P_I(i,g,r)**(1-esubd(i)))**(1/(1-esubd(i))))$CES(esubd(i)))

*       Cost function:

alias (i,i_);
$macro CY(g,r) ( \
  ( sum(i_, thetai(i_,g,r)*CI(i_,g,r)) + theta_f(g,r)*CF(g,r))$Leontief(esub(g)) + \
  (prod(i_, CI(i_,g,r)**thetai(i_,g,r))*CF(g,r)**theta_f(g,r))$CobbDouglas(esub(g)) + \
  ((sum(i_, thetai(i_,g,r)*CI(i_,g,r)**(1-esub(g))) + \
           theta_f(g,r)*CF(g,r)**(1-esub(g)))**(1/(1-esub(g))))$CES(esub(g)) )

prf_y(g,r)$vom(g,r)..           CY(g,r) =e= P(g,r) * (1-rto(g,r));

*       Demand functions:

$macro DDFM(i,g,r) (vdfm(i,g,r) * Y(g,r) * \
                        (CY(g,r)/CI(i,g,r))**esub(g) * \
                        (CI(i,g,r)/P_D(i,g,r))**esubd(i))$vdfm(i,g,r)
$macro DIFM(i,g,r) (vifm(i,g,r) * Y(g,r) * \
                        (CY(g,r)/CI(i,g,r))**esub(g) * \
                        (CI(i,g,r)/P_I(i,g,r))**esubd(i))$vifm(i,g,r)
$macro DFM(f,g,r)  (vfm(f,g,r)  * Y(g,r) * \
                        (CY(g,r)/CF(g,r))**esub(g) * \
                        (CF(g,r)/P_PF(f,g,r))**esubva(g))$vfm(f,g,r)

*       Associated tax revenue flows:

$macro REVTO(r)  (sum(g$vom(g,r),        rto(g,r)   * P(g,r)  * vom(g,r)*Y(g,r)))
$macro REVTFD(r) (sum((i,g)$vdfm(i,g,r), rtfd(i,g,r)* P(i,r)  * DDFM(i,g,r)))
$macro REVTFI(r) (sum((i,g)$vifm(i,g,r), rtfi(i,g,r)* PM(i,r) * DIFM(i,g,r)))
$macro REVTF(r)  (sum((f,g)$vfm(f,g,r),  rtf(f,g,r) * PF(f,r) * DFM(f,g,r)))
```

6.2.2 International transportation services – YT(j)

```
*       MPSGE :
*       $prod:YT(j)$vtw(j)  s:1
*               o:PT(j)         q:vtw(j)
*               i:P(j,r)        q:vst(j,r)

prf_yt(j)$vtw(j)..
        prod(r, P(j,r)**(vst(j,r)/vtw(j))) =e= PT(j);

*       Demand Function:

$macro DST(j,r)    (vst(j,r)*YT(j)*PT(j)/P(j,r))$vst(j,r)
```

6.2.3 Aggregate imports – M(i,r)

```
*       Profit function for bilateral trade aggregation:

* MPSGE :
* $prod:M(i,r)$vim(i,r) s:esubm(i)  s.tl:0
*       o:PM(i,r)       q:vim(i,r)
*       i:P(i,s)        q:vxmd(i,s,r)   p:pvxmd(i,s,r) s.tl:
*                a:RA(s) t:(-rtxs(i,s,r)) a:RA(r) t:(rtms(i,s,r)*(1-rtxs(i,s,r)))
*       i:PT(j)#(s)     q:vtwr(j,i,s,r) p:pvtwr(i,s,r) s.tl: a:RA(r) t:rtms(i,s,r)

*       User cost indices:

$macro P_M(i,s,r) ((P(i,s)*(1-rtxs(i,s,r))*(1+rtms(i,s,r))/pvxmd(i,s,r))$vxmd(i,s,r) + 1$(vxmd(i,s,r)=0))
$macro P_T(j,i,s,r) (PT(j)*(1+rtms(i,s,r))/pvtwr(i,s,r))$vtwr(j,i,s,r)

parameter       thetavxmd(i,s,r)        Value share of goods in imports,
                thetavtwr(j,i,s,r)      Value share of transportation services,
                thetam(i,s,r)           Bilateral import value share
                vxmt(i,s,r)             Value of imports gross transport cost;

vxmt(i,s,r)                     = vxmd(i,s,r)*pvxmd(i,s,r) + sum(j,vtwr(j,i,s,r)*pvtwr(i,s,r));
thetavxmd(i,s,r)$vxmt(i,s,r)    = vxmd(i,s,r)*pvxmd(i,s,r) / vxmt(i,s,r);
thetavtwr(j,i,s,r)$vxmt(i,s,r)  = vtwr(j,i,s,r)*pvtwr(i,s,r) / vxmt(i,s,r);
thetam(i,s,r)$vim(i,r)          = vxmt(i,s,r)/vim(i,r);

*       Price index of bilateral imports (Leontief cost function):

alias (j,j1);
$macro PT_M(i,s,r) (P_M(i,s,r)*thetavxmd(i,s,r) + sum(j1, P_T(j1,i,s,r)*thetavtwr(j1,i,s,r)))

*       Unit cost function for imports (CES):

alias (s,s_);
$macro CIM(i,r) ( \
   sum(s_, thetam(i,s_,r) * PT_M(i,s_,r) )$Leontief(esubm(i)) + \
   prod(s_, PT_M(i,s_,r)**thetam(i,s_,r) )$CobbDouglas(esubm(i)) + \
  (sum(s_, thetam(i,s_,r) * PT_M(i,s_,r)**(1-esubm(i)))**(1/(1-esubm(i))))$CES(esubm(i)) )
```

```
prf_m(i,r)$vim(i,r)..   CIM(i,r) =e= PM(i,r);

*       Demand function:

$macro DXMD(i,s,r)   ((vxmd(i,s,r)   * M(i,r) * (PM(i,r)/PT_M(i,s,r))**esubm(i))$vxmd(i,s,r))
$macro DTWR(j,i,s,r) ((vtwr(j,i,s,r) * M(i,r) * (PM(i,r)/PT_M(i,s,r))**esubm(i))$vtwr(j,i,s,r))

*       Associated tax revenue:

$macro REVTXS(r) (sum((i,s)$vxmd(i,r,s), -rtxs(i,r,s) * P(i,r) * dxmd(i,r,s)))

alias (j,j2);
$macro REVTMS(r) (sum((i,s)$vxmd(i,s,r), rtms(i,s,r) * \
        (P(i,s)*(1-rtxs(i,s,r))*DXMD(i,s,r) + sum(j2, PT(j2)*DTWR(j2,i,s,r)))))
```

6.2.4 Sluggish Factor Transformation – FT(f,r)

```
* MPSGE :
*       $prod:FT(sf,r)$evom(sf,r)  t:etrae(sf)
*               o:PS(sf,j,r)    q:vfm(sf,j,r)
*               i:PF(sf,r)      q:evom(sf,r)


parameter       thetavfm(f,j,r) Value shares of specific factors;
thetavfm(sf,j,r) = vfm(sf,j,r)/evom(sf,r);

alias (j,j3);
$macro PVFM(sf,r) (sum(j3,thetavfm(sf,j3,r)*PS(sf,j3,r)**(1+etrae(sf)))**(1/(1+etrae(sf))))

prf_ft(sf,r)$evom(sf,r)..       PF(sf,r) =e= PVFM(sf,r);
```

6.3 Market clearance

These equations do not appear explicitly in the MPSGE model, as they are generated automatically on the basis of the production function information provided above.

6.3.1 Firm output, public and private consumption, investment – PY(g,r)

```
mkt_p(g,r)$vom(g,r)..

        Y(g,r) * vom(g,r) =e= (RA(r)/P(g,r))$sameas(g,"C") +
                vom(g,r)$(sameas(g,"G") or sameas(g,"I")) +
```

6.3.2 Composite imports – PM(i,r)

```
mkt_pm(i,r)$vim(i,r)..  M(i,r) * vim(i,r) =e= sum(g, DIFM(i,g,r));
```

6.3.3 Transport services – PT(j)

```
mkt_pt(j)$vtw(j)..      YT(j) * vtw(j) =e= sum((i,s,r), DTWR(j,i,s,r));
```

6.3.4 Primary factors – PF(f,r)

```
mkt_pt(j)$vtw(j)..      YT(j) * vtw(j) =e= sum((i,s,r), DTWR(j,i,s,r));
```

6.3.5 Specific factors – PS(f,j,r)

```
mkt_pt(j)$vtw(j)..      YT(j) * vtw(j) =e= sum((i,s,r), DTWR(j,i,s,r));
```

6.4 Regional income balance – RA(r)

```
* MPSGE :
*       $demand:RA(r)
*       d:P("c",r)      q:vom("c",r)
*       e:P("c",rnum)   q:vb(r)
*       e:P("g",r)      q:(-vom("g",r))
*       e:P("i",r)      q:(-vom("i",r))
*       e:PF(f,r)       q:evom(f,r)

inc_ra(r)$(ra.lo(r) < ra.up(r))..
        RA(r) =e= sum(rnum, P("c",rnum)*vb(r))
                        - P("g",r)*vom("g",r)
                        - P("i",r)*vom("i",r)
                        + sum(f, PF(f,r)*evom(f,r))
                        + REVTO(r) + REVTFD(r) + REVTFI(r)
                        + REVTF(r) + REVTXS(r) + REVTMS(r);
```

6.5 Model Declaration and Benchmark Replication

```
model gtap7mcp /
        prf_y.Y,prf_m.M,prf_yt.YT,prf_ft.FT,
        mkt_p.P,mkt_pm.PM,mkt_pt.PT,mkt_pf.PF,mkt_ps.PS,
        inc_ra.RA/;

model gtap7cns /
        prf_y,prf_m,prf_yt,prf_ft,
        mkt_p,mkt_pm,mkt_pt,mkt_pf,mkt_ps,
        inc_ra/;

*       Assign default values:

Y.L(g,r) = 1;
M.L(i,r) = 1;
YT.L(j) = 1;
FT.L(sf,r) = 1;
P.L(g,r) = 1;
PM.L(j,r) = 1;
PT.L(j) = 1;
PF.L(f,r) = 1;
PS.L(sf,j,r) = 1;
RA.L(r) = vom("c",r);


*       Fix variables which should not enter the model:
```

```
Y.FX(g,r)$(vom(g,r)=0) = 1;
M.FX(i,r)$(vim(i,r)=0) = 1;
YT.FX(j)$(vtw(j)=0) = 1;
P.FX(j,r)$(vom(j,r)=0) = 1;
PM.FX(j,r)$(vim(j,r)=0) = 1;
PT.FX(j)$(vtw(j)=0) = 1;
PF.FX(f,r)$(evom(f,r)=0) = 1;
PS.FX(f,j,r)$((not sf(f)) or (vfm(f,j,r)=0)) = 1;
FT.FX(f,r)$((not sf(f)) or (evom(f,r)=0)) = 1;

*       Establish a price normalization using the reference region:

RA.FX(rnum) = RA.L(rnum);

gtap7mcp.iterlim = 0;
solve gtap7mcp using mcp;

gtap7mcp.iterlim = 10000;

*       Verify benchmark consistency with both MCP and CNS models:

gtap7cns.iterlim = 0;
solve gtap7cns using cns;
gtap7cns.iterlim = 10000;

*       Run a GFT scenario using the MCP model;

rtxs(i,r,s) = 0;
rtms(i,r,s) = 0;

*       Need to remove bounds on Y and M to solve this as a CNS:
Y.LO(g,r) = -inf;
M.LO(i,r) = -inf;
solve gtap7cns using cns;

*       Verify consistency with the MCP model:
gtap7mcp.iterlim = 0;
solve gtap7mcp using mcp;
```

6.6 Declaration and Replication – MPSGE Version

```
$title  Read GTAP 7 Basedata and Replicate the Benchmark in MPSGE

$if not set ds $set ds test

$include gtap7data

parameter
        esub(g)                 Top-level elasticity indemand /C 1/;
display esub;
display  esubva;
display esubd;


$ontext
```

```
$model:gtap7

$sectors:
        Y(g,r)$vom(g,r)         ! Supply
        M(i,r)$vim(i,r)         ! Imports
        YT(j)$vtw(j)            ! Transportation services
        FT(f,r)$(sf(f) and evom(f,r))   ! Specific factor transformation

$commodities:
        P(g,r)$vom(g,r)         ! Domestic output price
        PM(j,r)$vim(j,r)        ! Import price
        PT(j)$vtw(j)            ! Transportation services
        PF(f,r)$evom(f,r)       ! Primary factors rent
        PS(f,g,r)$(sf(f) and vfm(f,g,r))        ! Sector-specific primary factors

$consumers:
        RA(r)                   ! Representative agent

$prod:Y(g,r)$vom(g,r)   s:esub(g)    i.tl:esubd(i)  va:esubva(g)
        o:P(g,r)        q:vom(g,r)      a:RA(r)  t:rto(g,r)
        i:P(i,r)        q:vdfm(i,g,r)   p:(1+rtfd0(i,g,r)) i.tl:  a:RA(r) t:rtfd(i,g,r)
        i:PM(i,r)       q:vifm(i,g,r)   p:(1+rtfi0(i,g,r)) i.tl:  a:RA(r) t:rtfi(i,g,r)
        i:PS(sf,g,r)    q:vfm(sf,g,r)   p:(1+rtf0(sf,g,r))  va:    a:RA(r) t:rtf(sf,g,r)
        i:PF(mf,r)      q:vfm(mf,g,r)   p:(1+rtf0(mf,g,r))  va:    a:RA(r) t:rtf(mf,g,r)

$prod:YT(j)$vtw(j)  s:1
        o:PT(j)         q:vtw(j)
        i:P(j,r)        q:vst(j,r)

$prod:M(i,r)$vim(i,r)   s:esubm(i)  s.tl:0
        o:PM(i,r)       q:vim(i,r)
        i:P(i,s)        q:vxmd(i,s,r)   p:pvxmd(i,s,r) s.tl:
+               a:RA(s) t:(-rtxs(i,s,r)) a:RA(r) t:(rtms(i,s,r)*(1-rtxs(i,s,r)))
        i:PT(j)#(s)     q:vtwr(j,i,s,r) p:pvtwr(i,s,r) s.tl:
+               a:RA(r) t:rtms(i,s,r)

$prod:FT(sf,r)$evom(sf,r)  t:etrae(sf)
        o:PS(sf,j,r)    q:vfm(sf,j,r)
        i:PF(sf,r)      q:evom(sf,r)

$demand:RA(r)
        d:P("c",r)      q:vom("c",r)
        e:P("c",rnum)   q:vb(r)
        e:P("g",r)      q:(-vom("g",r))
        e:P("i",r)      q:(-vom("i",r))
        e:PF(f,r)       q:evom(f,r)

$offtext
$sysinclude mpsgeset gtap7

gtap7.iterlim = 0;
$include gtap7.gen
solve gtap7 using mcp;
```

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Table 6: Commodities and Industries

PDR	Paddy rice	LUM	Wood products
WHT	Wheat	PPP	Paper products, publishing
GRO	Cereal grains nec	P_C	Petroleum, coal products
V_F	Vegetables, fruit, nuts	CRP	Chemical,rubber,plastic prods
OSD	Oil seeds	NMM	Mineral products nec
C_B	Sugar cane, sugar beet	I_S	Ferrous metals
PFB	Plant-based fibers	NFM	Metals nec
OCR	Crops nec	FMP	Metal products
CTL	Cattle,sheep,goats,horses	MVH	Motor vehicles and parts
OAP	Animal products nec	OTN	Transport equipment nec
RMK	Raw milk	ELE	Electronic equipment
WOL	Wool, silk-worm cocoons	OME	Machinery and equipment nec
FRS	Forestry	OMF	Manufactures nec
FSH	Fishing	ELY	Electricity
COA	Coal	GDT	Gas manufacture, distribution
OIL	Oil	WTR	Water
GAS	Gas	CNS	Construction
OMN	Minerals nec	TRD	Trade
CMT	Meat: cattle,sheep,goats,horse	OTP	Transport nec
OMT	Meat products nec	WTP	Sea transport
VOL	Vegetable oils and fats	ATP	Air transport
MIL	Dairy products	CMN	Communication
PCR	Processed rice	OFI	Financial services nec
SGR	Sugar	ISR	Insurance
OFD	Food products nec	OBS	Business services nec
B_T	Beverages and tobacco products	ROS	Recreation and other services
TEX	Textiles	OSG	PubAdmin/Defence/Health/Educat
WAP	Wearing apparel	DWE	Dwellings
LEA	Leather products	CGD	Aggregate investment

Table 7: Primary Factors

Mobile factors:		
	SKL	Skilled labor
	LAB	Unskilled labor
	CAP	Capital
"Sluggish" factors:		
	LND	Land
	RES	Natural resources